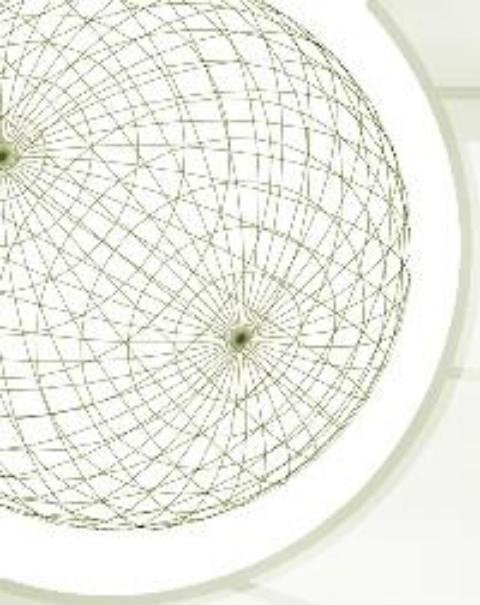


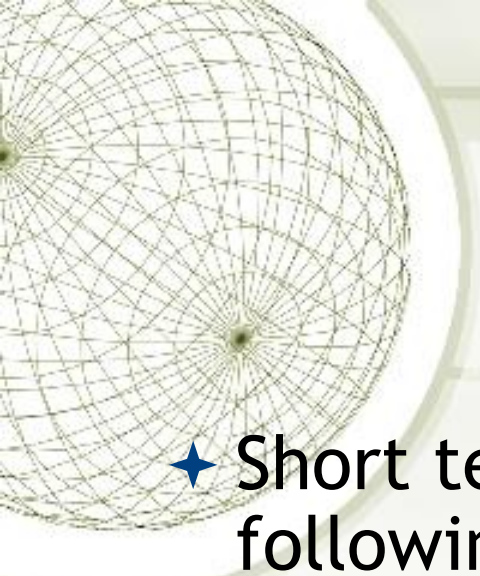


Traffic Simulation

Josh Gilkerson

Wei Li

David Owen



Uses

- ★ Short term forecasting to determine actions following an incident that changes the roadway.
- ★ Anticipatory guidance for Advanced Traveler Information Systems (ATIS) to help drivers make better decisions.
- ★ Determination of how to spend money on improving infrastructure.
- ★ Planning for closures/construction.



Safety Modeling

- ★ Developing safety predictions is desirable.
- ★ Ignored by most models at present.
- ★ Difficult to predict human error.
- ★ Difficult to add more vulnerable users of the road.
 - ★ Cyclists
 - ★ Pedestrians



Modeling Approaches

- ★ Scope

- ★ Micro

- ★ Macro

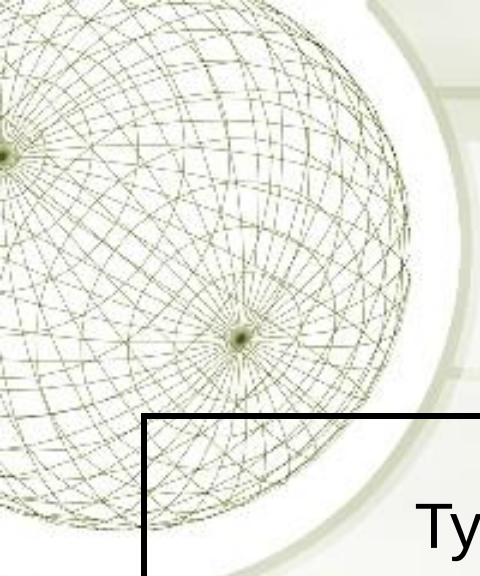
- ★ Meso

- ★ Discrete vs. Continuous

- ★ Situations

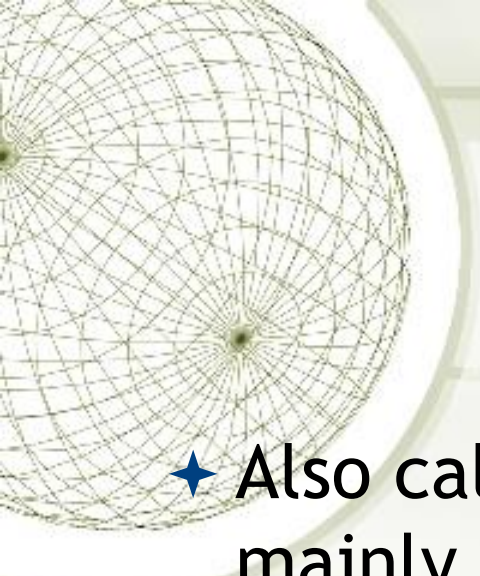
- ★ Intersections

- ★ Freeways



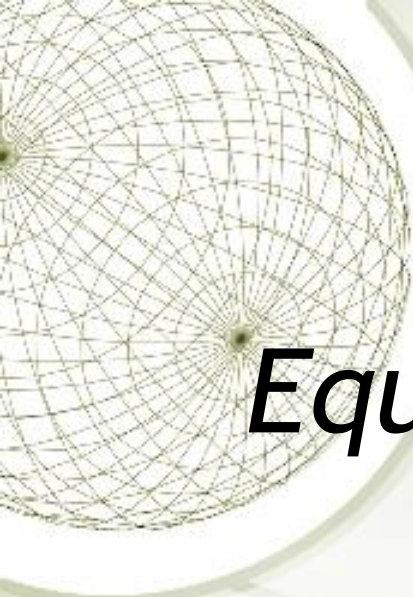
Popularity

Type of Simulation	Number of Packages
Microscopic	65
Mesososcopic	3
Macroscopic	16



Macroscopic Traffic Simulation

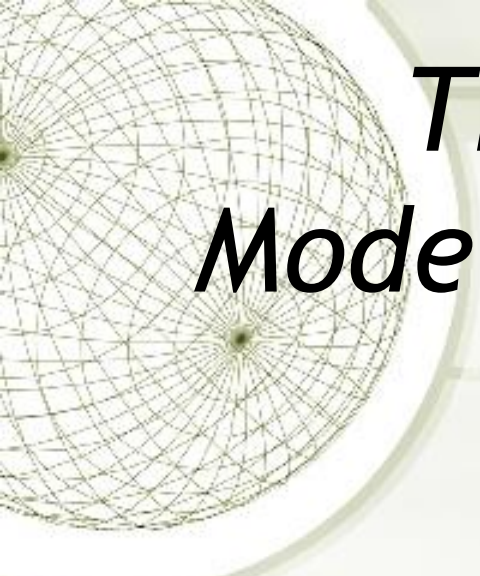
- ★ Also called continuous flow simulation, mainly used in traffic flow analysis
- ★ Originated from the late 1960's and the early 1970's
 - ★ British TRANSYT Program
 - ★ Simulation of urban arterial traffic signal control
 - ★ American FREQ Program, FREFLO Program
 - ★ Motorway applications



Traditional Mathematical Modeling: Continuity Equation for Vehicle Density

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho V)}{\partial x} = \pm v(x, t)$$

- ★ Number of vehicles is conserved
- ★ Vehicle density per lane at position x and time t - $\rho(x, t)$
- ★ Average vehicle velocity - $v(x, t)$



Traditional Mathematical Modeling: Dynamical Velocity Equation

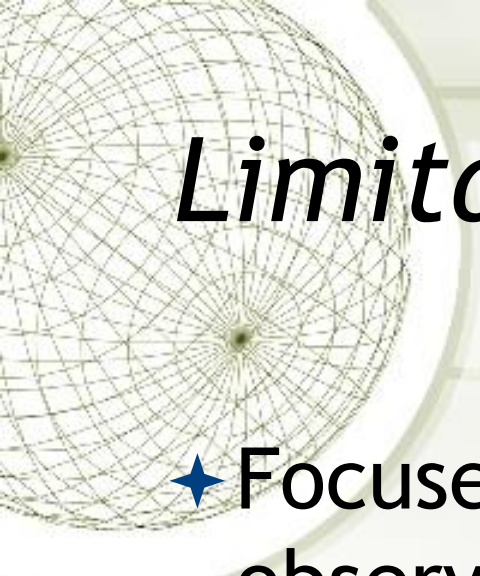
$$\frac{\partial V}{\partial t} + V \frac{\partial V}{\partial x} = -\frac{1}{\rho} \frac{\partial P}{\partial x} + \frac{1}{\tau} (V_e - V)$$

- ✦ The change of the average vehicle velocity depends on 3 terms
- ✦ Transport term - propagation of the velocity profile with the velocity of the vehicles
- ✦ Pressure term - anticipation of spatial changes in the traffic situation, or dispersion effects due to a finite variance of the vehicle velocities
- ✦ Relaxation term - adaptation to a dynamic equilibrium velocity with relaxation time



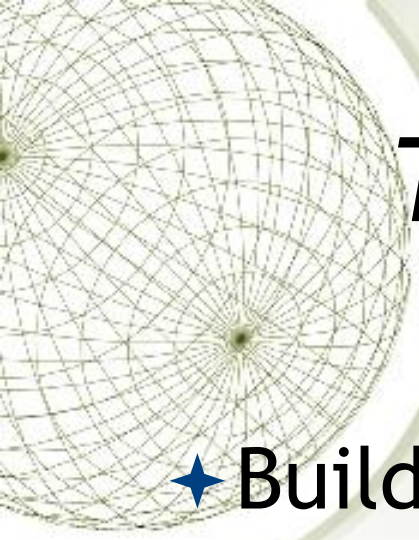
Characteristics of the Congested Traffic

- ★ Traffic jam is independent of the initial conditions and the spatially averaged density
 - ★ Outflow from traffic jams is 1800 ± 200 vehicles per kilometer and lane
 - ★ Dissolution velocity is -15 ± 5 kilometers per hour
- ★ Related to the special motion pattern of the traffic jams
 - ★ Outflow is related to the time interval between successive departures from the traffic jam
 - ★ Therefore independent of the type and density of congested traffic
 - ★ The dissolution velocity of traffic jams is nearly constant



Limitations of the Traditional Model

- ★ Focuses on reproducing the empirically observed flow-density relation and the regime of unstable traffic flow
- ★ Unable to describe the observed spectrum of non-linear phenomena and their characteristic properties



The Non-local Gas-Kinetic Traffic Model

- ★ Builds upon the above traffic congestion characteristics
- ★ Doesn't have the limitation of the traditional model
- ★ Derived from microscopic models of driver-vehicle behavior

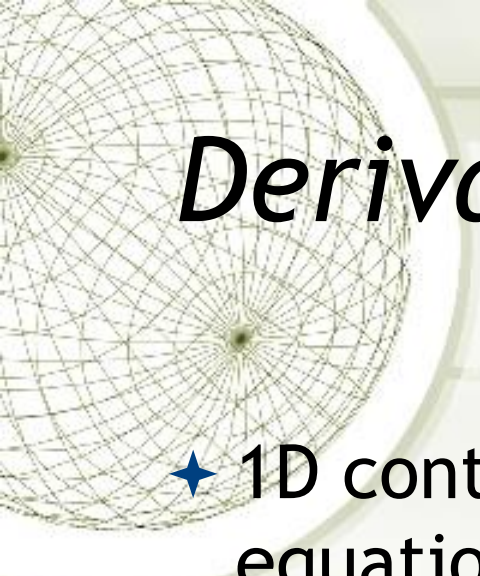
Derivation of the Underlying Gas-Kinetic Equation

- ★ The kinetic equation of the evolution of the coarse-grained phase-space density
- ★ The microscopic dynamics of individual driver-vehicle units
- ★ The kinetic evolution equation for the phase-space density is derived by partial integration

$$\tilde{\rho}(x, v, t) = \sum_a \int dt' \int dx' \int dv' g(t - t', x - x', v - v') \\ \times \delta(x' - x_a(t)) \delta(v' - v_a(t))$$

$$\frac{dx_\alpha}{dt} = v_\alpha$$
$$\frac{dv_\alpha}{dt} = \frac{v_\alpha^0 - v_\alpha}{\tau_\alpha} - \sum_{\beta \neq \alpha} f_\beta \beta$$

$$\frac{\partial \tilde{\rho}}{\partial t} + \frac{\partial}{\partial x}(\tilde{\rho} v) + \frac{\partial}{\partial v} \left[\tilde{\rho} \frac{V_0 - v}{\tau} \right] = \frac{\partial}{\partial v}(\tilde{\rho} f_{\text{int}}) + \frac{\partial^2}{\partial v^2}(\tilde{\rho} D)$$

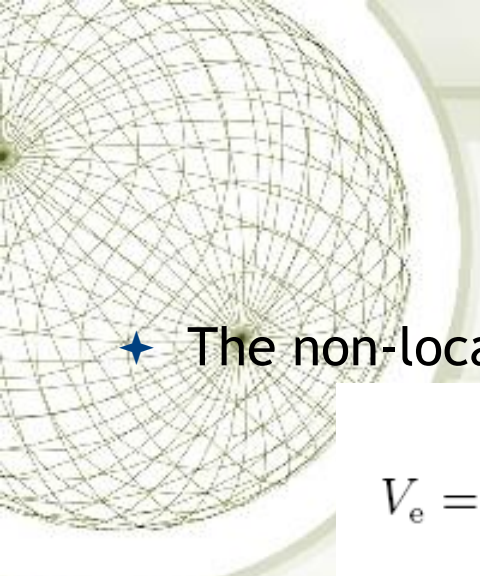


Derivation of the Macroscopic Equations

- ★ 1D continuity equation (The number of vehicles is fixed)
- ★ Dynamical velocity equation with non-local term

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho V)}{\partial x} = 0$$

$$\left(\frac{\partial}{\partial t} + V \frac{\partial}{\partial x}\right)V = -\frac{1}{\rho} \frac{\partial(\rho A V^2)}{\partial x} + \frac{V_0 - V}{\tau} - \frac{V_0 A(\rho)}{\tau A(\rho_{\max})} \left(\frac{\rho_a T V}{1 - \rho_a / \rho_{\max}}\right)^2 B(\delta_V)$$



Analytic Solution

- ★ The non-local dynamical equilibrium velocity

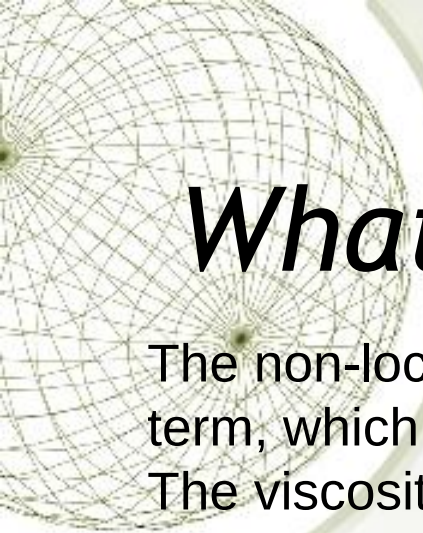
$$V_e = V_0 \left[1 - \frac{\theta + \theta'}{2A(\rho_{\max})} \left(\frac{\rho' T}{1 - \rho' / \rho_{\max}} \right)^2 B(\delta_V) \right] .$$

- ★ Boltzmann factor

$$B(\delta_V) = 2 \left[\delta_V \frac{e^{-\delta_V^2/2}}{\sqrt{2\pi}} + (1 + \delta_V^2) \int_{-\infty}^{\delta_V} dy \frac{e^{-y^2/2}}{\sqrt{2\pi}} \right]$$

- ★ Intra-lane variance approximated by the constitutive relation

$$\theta(x, t) = A(\rho) V^2(x, t) = \left[A_0 + \Delta A \tanh \left(\frac{\rho(x, t) - \rho_c}{\Delta \rho} \right) \right] V^2(x, t) ,$$



What's new in the New Model

The non-local Gas-Kinetic traffic model has the extra non-local braking term, which is similar to a viscosity term

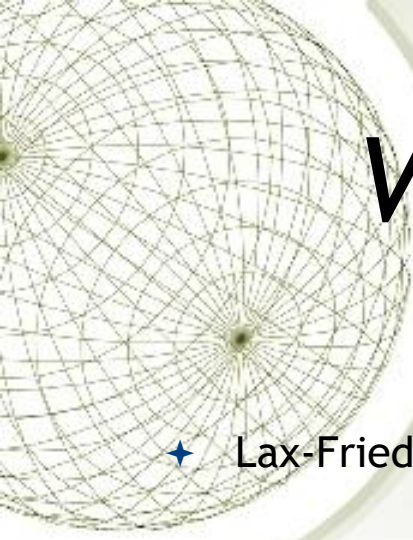
The viscosity term results in unphysical humps in the vehicle density, while the non-local braking term does not

We need to solve the following equation numerically

$$\frac{\partial u}{\partial t} + \frac{\partial f(u)}{\partial x} = s(u).$$

A variety of numerical standard methods developed for hydrodynamic problems can be used here

Good numerical stability and integration speed; real time simulation doesn't need super computer to do the calculation



Various Explicit Numerical Methods

- ★ Lax-Friedrichs method

$$u_j^{n+1} = \frac{u_{j-1}^n + u_{j+1}^n}{2} - \frac{\Delta t}{2\Delta x}(f_{j+1}^n - f_{j-1}^n) + \Delta t s_j^n.$$

- ★ Upwind method

$$u_j^{n+1} = u_j^n - \frac{\Delta t}{\Delta x}(f_j^n - f_{j-1}^n) + \Delta t s_j^n.$$

- ★ MacCormack method

$$\tilde{u}_j^n = u_j^n - \frac{\Delta t}{\Delta x}(f_j^n - f_{j-1}^n) + \Delta t s_j^n, \quad (\text{predictor})$$

$$u_j^{n+1} = \frac{1}{2} \left[\tilde{u}_j^n + u_j^n - \frac{\Delta t}{\Delta x}(\tilde{f}_{j+1}^n - \tilde{f}_j^n) + \Delta t \tilde{s}_j^n \right]. \quad (\text{corrector})$$

- ★ Lax-Wendroff method

$$u_{j+\frac{1}{2}}^{n+\frac{1}{2}} = \frac{1}{2} \left[u_j^n + u_{j+1}^n - \frac{\Delta t}{\Delta x}(f_{j+1}^n - f_j^n) + \frac{\Delta t}{2}(s_j^n + s_{j+1}^n) \right] \quad (\text{predictor})$$

$$u_j^{n+1} = u_j^n - \frac{\Delta t}{\Delta x} \left[f_{j+\frac{1}{2}}^{n+\frac{1}{2}} - f_{j-\frac{1}{2}}^{n+\frac{1}{2}} \right] + \frac{\Delta t}{2} \left[s_{j+\frac{1}{2}}^{n+\frac{1}{2}} + s_{j-\frac{1}{2}}^{n+\frac{1}{2}} \right]. \quad (\text{corrector})$$



Initial and Boundary Conditions

- ★ Dirichlet boundary conditions

- ★ *Fixed $u(0, t)$ and $u(L, t)$*

- ★ Homogeneous von Neumann boundary conditions

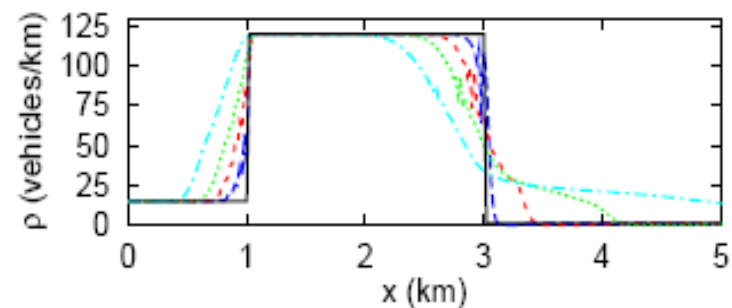
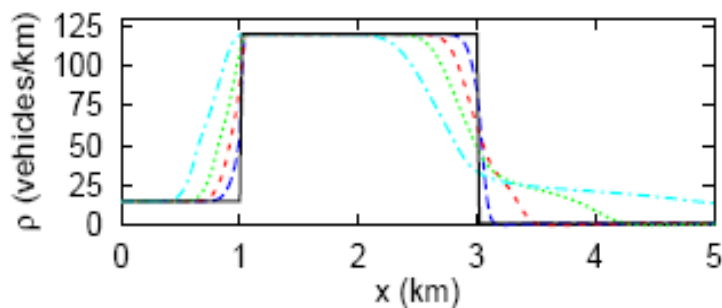
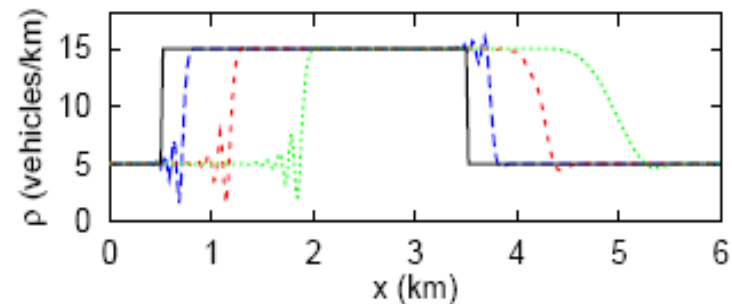
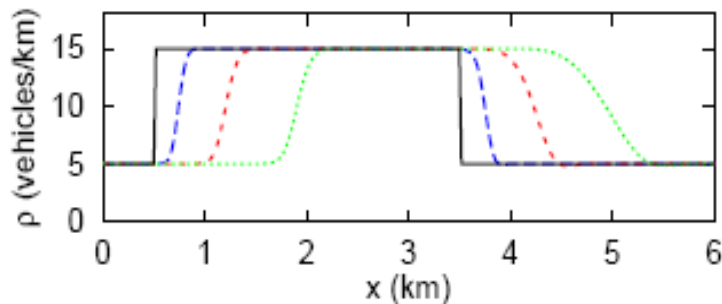
$$\frac{\partial u(0, t)}{\partial x} = 0, \quad \frac{\partial u(L, t)}{\partial x} = 0.$$

- ★ Free boundary conditions

$$\frac{\partial^2 u(0, t)}{\partial x^2} = \frac{\partial^2 u(L, t)}{\partial x^2} = 0$$

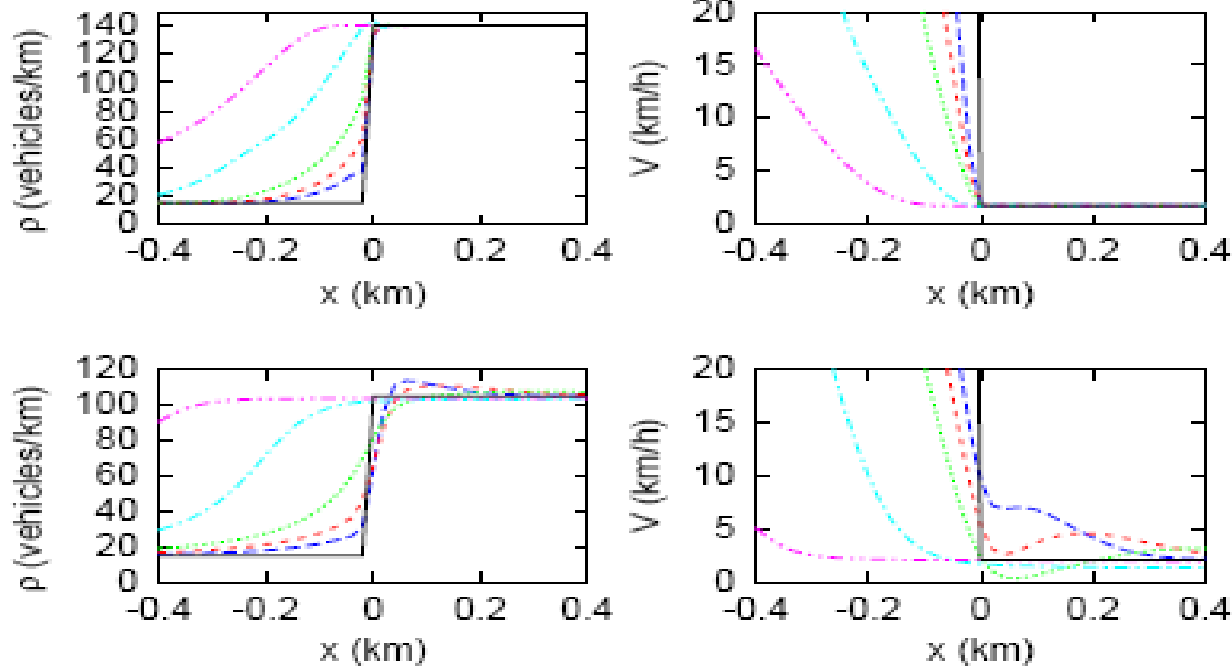
Comparison of the Numerical Solutions

- Comparison between the Upwind method and the MacCormack method: simulations of the non-local gas-kinetic-based traffic model with discontinuous initial conditions



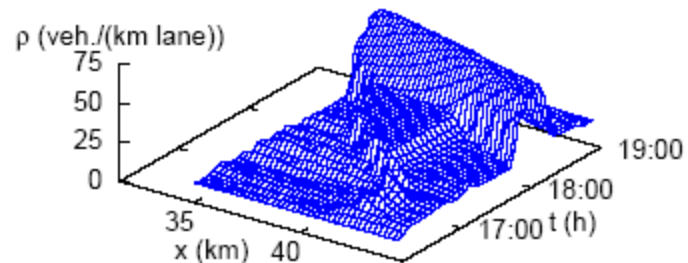
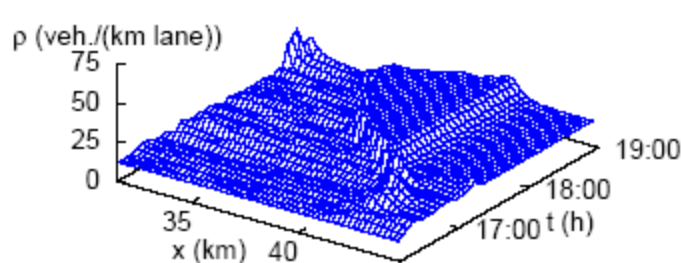
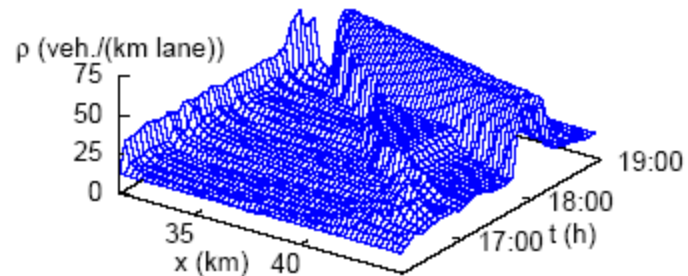
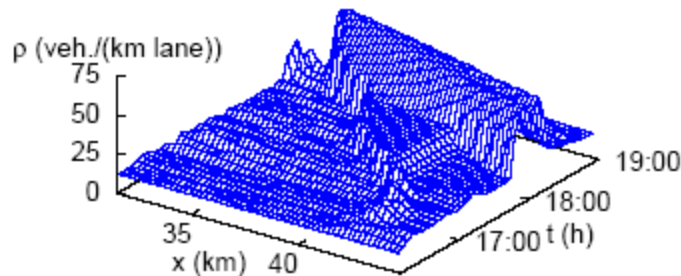
Comparison with the Traditional Model

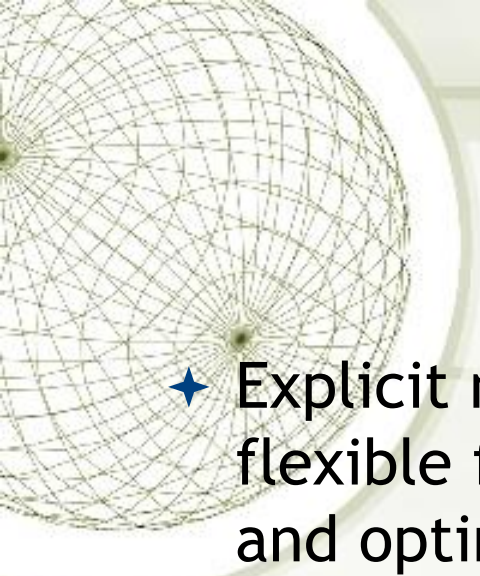
- ★ First stages of the density and velocity profiles evolving from a discontinuous upstream front



Numerical Solutions

- ★ Simulation with different empirical boundary conditions at the German freeway A8 near Munich,





Conclusions

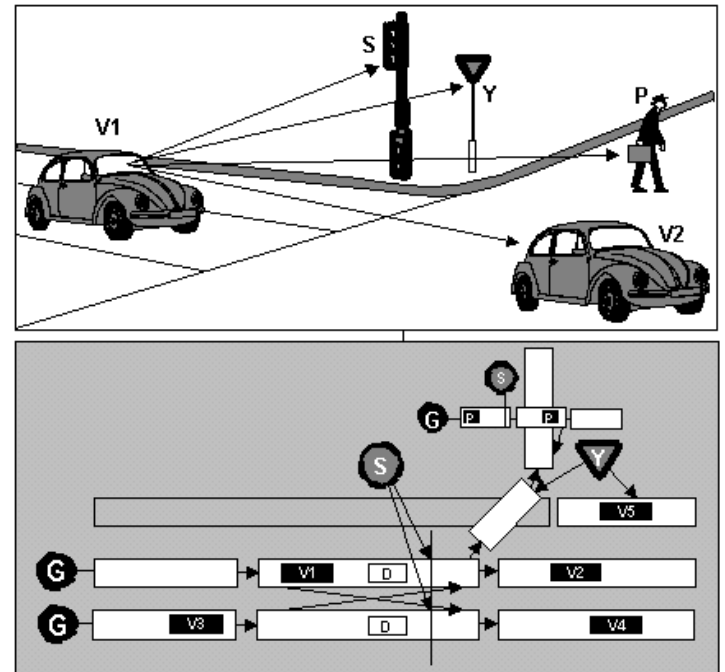
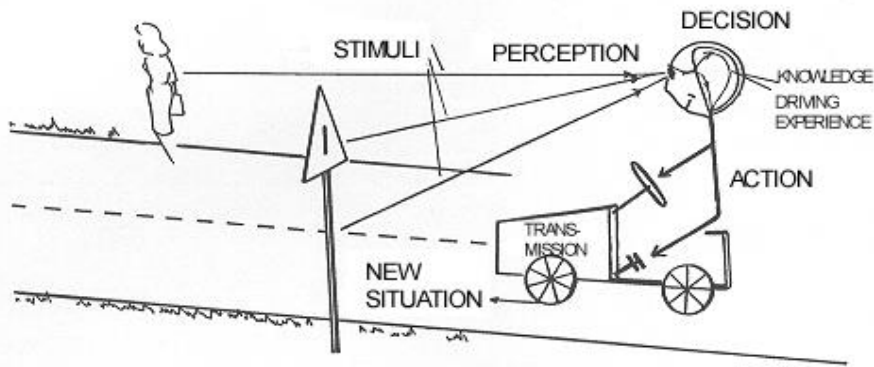
- ✦ Explicit methods are less robust, but much more flexible for time-dependent boundary conditions and optimization problems
- ✦ The upwind method is more accurate than the Lax-Friedrichs method among the explicit first-order methods
- ✦ The second-order MacCormack and the Lax-Wendroff methods are slower and produce unrealistic oscillations close to steep gradients
- ✦ The simulation of the non-local gas-kinetic-based traffic model is much more efficient than the models with viscosity or diffusion terms



Microscopic Traffic Simulation

- ★ Unlike Macroscopic simulation, every vehicle in Microscopic model is simulated.
- ★ There are three behaviors:
 - ★ Accelerations
 - ★ Braking decelerations
 - ★ Lane changes
- ★ In order to achieve accuracy in modeling the traffic, many factors must be considered. This leads to a simulation model with high degree of parameters (50 parameters model is common).

External Factors



Intelligent Driver Model (IDM)

This model simulates single-lane main road and simple lane-change model for the on-ramps.

There are seven parameters involved:

Parameter	Typical value
Desired velocity v_0	120 km/h
Safe time headway T	1.5 s
Maximum acceleration a	1 m/s ²
Comfortable deceleration b	2 m/s ²
Minimum distance s_0	2 m
Jam distance s_1	0 m
Acceleration exponent δ	4

TABLE I. Parameters of the IDM used as reference values.



IDM Acceleration

Acceleration governs how each individual vehicle moves around the roads.

IDM acceleration is a continuous function of its own velocity v , spatial gap to the leading vehicle s , and velocity difference Δv .

$$\dot{v} = a \left[1 - \left(\frac{v}{v_0} \right)^\delta - \left(\frac{s^*(v, \Delta v)}{s} \right)^2 \right]$$

This expression gives us the ability to express the tendency to accelerate faster when the road is free

$$a \left[1 - (v/v_0)^\delta \right]$$

and the tendency to decelerate when the vehicle comes too close to the one in front of it.

$$-a(s^*/s)^2$$



IDM Acceleration (cont.)

The deceleration depends on s^* which is the “desired minimum gap”.

$$s^*(v, \Delta v) = s_0 + s_1 \sqrt{\frac{v}{v_0}} + Tv + \frac{v\Delta v}{2\sqrt{ab}}$$

This varies according to v and Δv from vehicle to vehicle.



IDM Model Properties

With the underlying model, the following behavior can be achieved:

1. Nearly empty freeway

Characterized by

$$s \gg v_0 T$$

The acceleration is given by

$$\dot{v} = a[1 - (v/v_0)^\delta]$$

The vehicle accelerates with maximum acceleration allowed by a .

The acceleration coefficient δ affects how the acceleration changes when it approaches v_0 . When $\delta = 1$, we have exponential approach, but when δ is very large, it is constant $\dot{v} = a$ and drops to 0 when it reach v_0



IDM Model Properties (cont)

2. Dense equilibrium traffic

Characterized by $\dot{v} = 0$ and $v < v_0/2$.

Each vehicle follows each other with constant distance

$$s_e(v) \approx s^*(v, 0) = s_0 + s_1 \sqrt{(v/v_0)} + vT$$

s_0 denotes the minimum bumper-to-bumper distance between vehicles.

3. Approaching standing obstacle

Characterized by $\Delta v = -v$ and $s \gg v^2/(2b)$

The vehicles will decelerate in a way that the comfortable deceleration b will not be exceeded.

4. Emergency situation

Characterized by $s < (\Delta v)^2/(2b)$.

The driver tries to keep the vehicle under control. This can be done by adding a higher deceleration value.

IDM Results

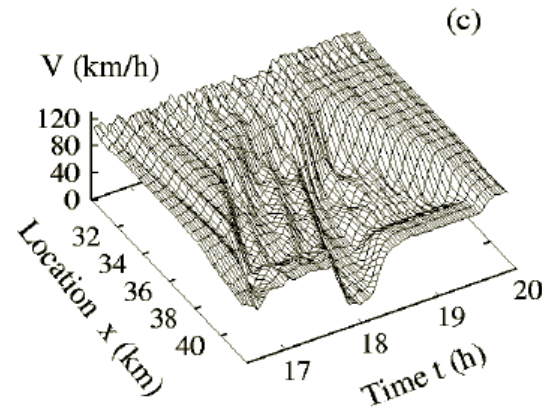
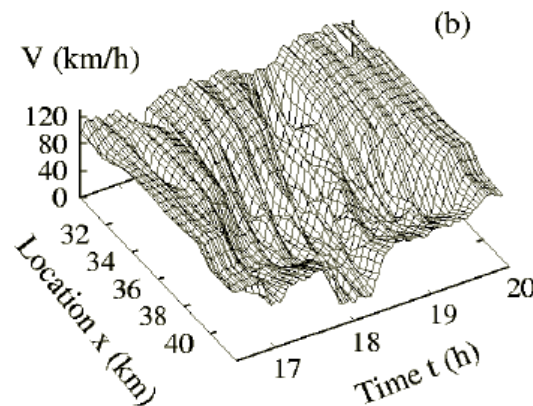
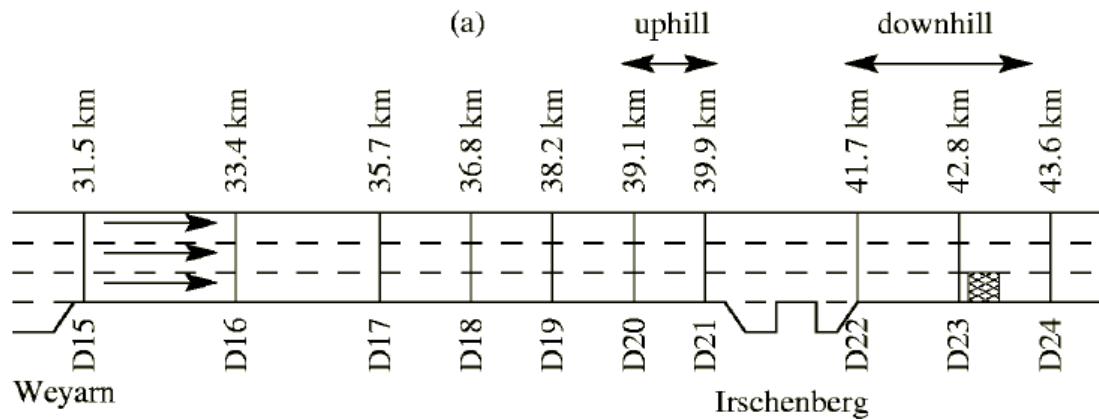


FIG. 1. Traffic breakdown at a section of the A8-East used as test szenario. (a) Sketch of the section. The square downstream of D 23 indicates a temporary closing of the right lane due to an incident (see text). (b) Measured and (c) simulated locally averaged velocity.

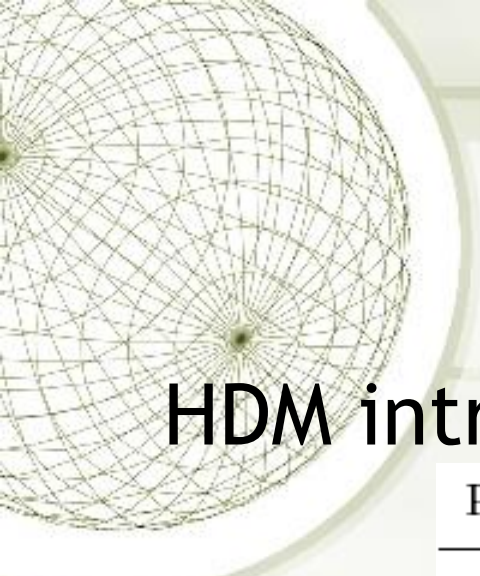


Human Driver Model (HDM)

Even though IDM is “intelligent” enough (in a sense of acceleration/deceleration behavior) there are many other factors which can be extended through this model.

HDM extended behaviors:

- ✦ Finite reaction time.
- ✦ Estimation errors.
- ✦ Temporal anticipation.
- ✦ Spatial anticipation.
- ✦ Adaptation to the global traffic situation.



HDM Parameters

HDM introduces the following parameters

Parameter	value
Reaction time T'	0 s - 1.6 s
Update time interval Δt	0.01 s - 2.5 s
Number of anticipated vehicles n_a	1 - 8
Relative distance error V_s	5%
Inverse TTC error r_c	0.01/s
error correlation times $\tau_s, \tau_{\Delta v}$	20 s
Traffic adaptation factor β_a	0.5
Traffic adaptation factor β_T	1.3
Traffic adaptation time τ	120 s



General Model

We restrict HDM to a single-lane dynamics (such as IDM). The consideration is the acceleration with the following general form:

$$\frac{dv_{\alpha}}{dt} = a^{\text{mic}}(s_{\alpha}, v_{\alpha}, \Delta v_{\alpha})$$

Where

- v_{α} Its own velocity.
- s_{α} Net distance.
- Δv_{α} Velocity difference with leading vehicle.

The characteristics of this model are:

- ★ Instantaneous reaction.
- ★ Reaction to immediate predecessor/successor.
- ★ Exact estimating ability of the driver.
- ★ Acceleration is determined by local traffic environment.



Finite Reaction time

The time it takes for a driver to response to his environment.

Reaction time T' is implemented by evaluating a^{mic} at time $t - T'$.
However, when T' is not a multiple of the update time interval, we will use bilinear interpolation according to:

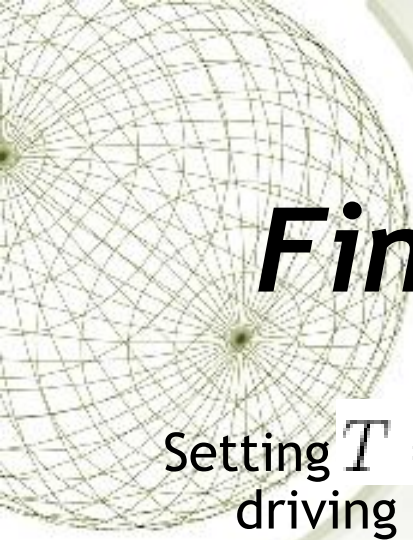
$$x(t - T') = \beta x_{t-n-1} + (1 - \beta)x_{t-n}$$

Where

x denotes a^{mic}

x_{t-n} denotes x evaluated at $n = \text{floor}(T'/\Delta t)$ time steps before the current one.

The weight factor is $\beta = T'/\Delta t - n$

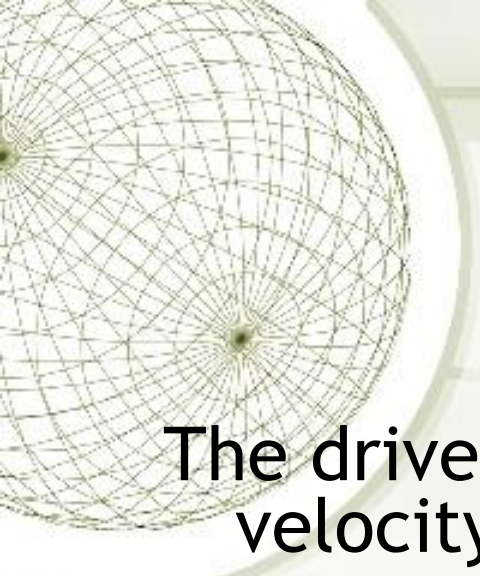
A decorative wireframe sphere is located in the top-left corner of the slide.

Finite Reaction time (cont)

Setting $T = T'$ achieves the effects of lower limit of safe driving only for the following worst-case scenario:

- ✦ The preceding vehicle suddenly brakes at maximum deceleration.
- ✦ The velocities of the leading and following vehicles are the same.
- ✦ The maximum decelerations are the same.
- ✦ No multi-anticipation.

In reality T depends on driving style while T' depends on physiological parameters (weakly correlated).



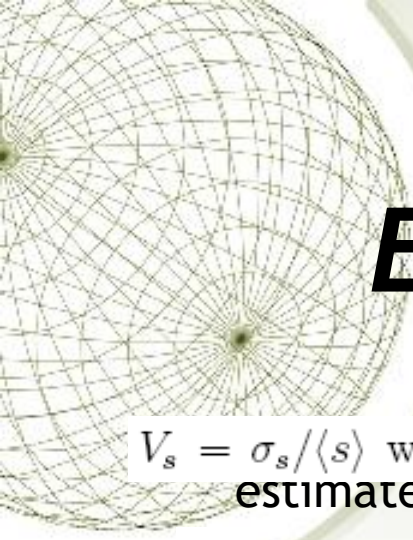
Estimation errors

The driver cannot exactly estimate the velocity of the other vehicles. Thus, the error must be simulated.

The following is a nonlinear stochastic formula for estimating distance and velocity difference.

$$s^{\text{est}}(t) = s(t) \exp(V_s w_s(t))$$

$$(\Delta v)^{\text{est}}(t) = \Delta v + s(t) r_c w_{\Delta v}(t).$$



Estimation errors (cont.)

$V_s = \sigma_s / \langle s \rangle$ with $\sigma_s^2 = \langle (s - \langle s \rangle)^2 \rangle$ is the variation coefficient of the estimate.

r_c is the inverse TTC as measure of error in Δv .

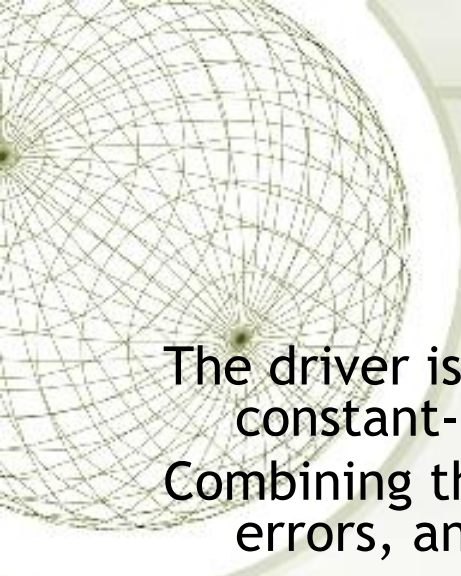
$w_s(t)$ and $w_{\Delta v}(t)$ obey independent Wiener processes $w(t)$ with $\tau = \tau_s$ and $\tau_{\Delta v}$ correlation time respectively.

$w(t)$ is defined such that:

$$\frac{dw}{dt} = -\frac{w}{\tau} + \xi(t).$$

With

$$\langle \xi \rangle = 0, \quad \langle \xi(t) \xi(t') \rangle = \frac{2}{\tau} \delta(t - t')$$



Temporal anticipation

The driver is able to anticipate the future velocity by using constant-acceleration heuristic.

Combining the knowledge of finite reaction time, estimation errors, and temporal anticipation, we have the following:

$$\frac{dv}{dt} = a^{\text{mic}}(s'_\alpha, v'_\alpha, \Delta v'_\alpha),$$

with

$$s'_\alpha(t) = \left[s_\alpha^{\text{est}} - T' \Delta v_\alpha^{\text{est}} \right]_{t-T'},$$

$$v'_\alpha(t) = \left[v_\alpha^{\text{est}} + T' a_\alpha \right]_{t-T'},$$

and

$$\Delta v'_\alpha(t) = \Delta v_\alpha^{\text{est}}(t - T').$$



Spatial anticipation

The driver is able to anticipate due to observation of several vehicles ahead.

For this HDM splits the acceleration model into two parts:

- ★ Single vehicle acceleration on empty road.
- ★ Vehicle-vehicle interaction with preceding vehicle.

$$a^{\text{mic}}(s_\alpha, v_\alpha, \Delta v_\alpha) := a_\alpha^{\text{free}} + a^{\text{int}}(s_\alpha, v_\alpha, \Delta v_\alpha)$$

We model the reaction to several vehicles ahead by summing up the vehicle-vehicle interactions

$a_{\alpha\beta}^{\text{int}}$ from vehicle β to vehicle α for the n_a nearest preceding vehicles.

$$\frac{d}{dt}v_\alpha(t) = a_\alpha^{\text{free}} + \sum_{\beta=\alpha-n_a}^{\alpha-1} a_{\alpha\beta}^{\text{int}},$$

Where

$$a_{\alpha\beta}^{\text{int}} = a^{\text{int}}(s_{\alpha\beta}, v_\alpha, v_\alpha - v_\beta)$$

And

$$s_{\alpha\beta} = \sum_{j=\beta+1}^{\alpha} s_j$$



Adaptation to the global traffic situation

Human drivers remember when they got stuck in a congested traffic for hours. HDM models this by applying 'level-of-service' to the traffic.

$$\lambda = v_\alpha / v_0$$

When a driver encounters traffic with low λ , drivers gradually change their driving style from 'free-traffic-mode' to 'congested-traffic-mode'.

This change involves the gradual change on the underlying model parameters as a new, slowly varying variable $\lambda(t)$

In IDM specifically, we change and with the following

$$a(\lambda) = a[1 + (1 - \lambda)(\beta_a - 1)]$$

$$T(\lambda) = T[1 + (1 - \lambda)(\beta_T - 1)]$$

Current simulation software

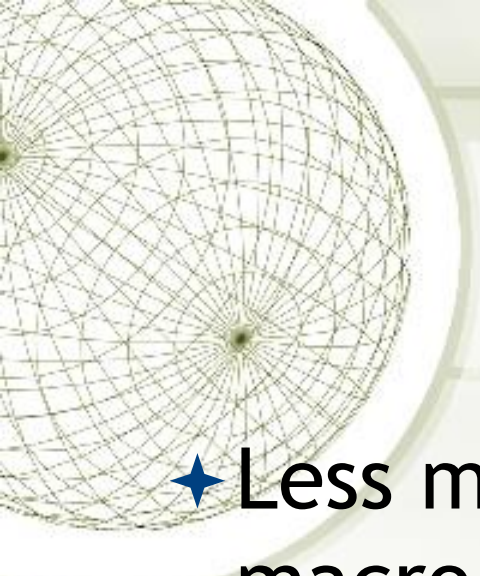
Halcrow's AIMSUN and VISSIM



Dubai Spine Road study – VISSIM model

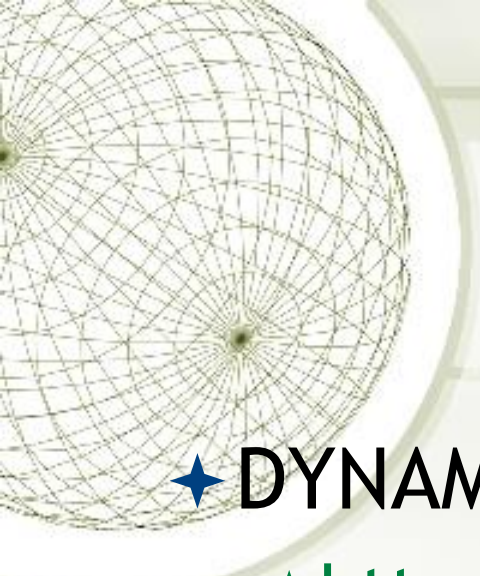


Otley Road, Leeds - AIMSUN model



Mesosopic Simulation

- ★ Less mature than either micro- or macro-scale methods
- ★ Tries to combine the advantages of both
 - ✦ Detail (microscale)
 - ✦ Scalability to larger networks (macroscale)



Mesosopic Packages

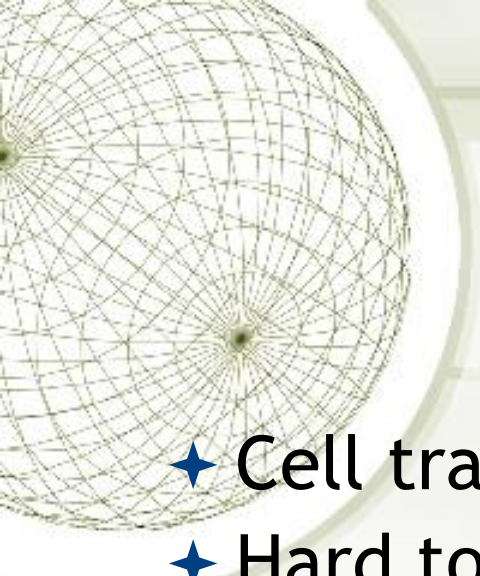
- ★ DYNAMIT

- ★ <http://mit.edu/its/dynamit.html>

- ★ DYNEMO

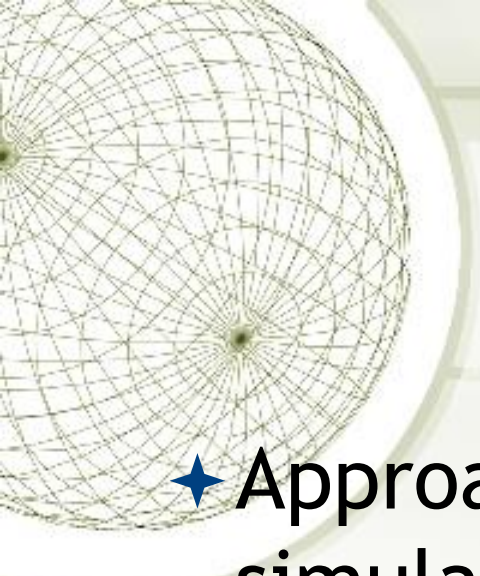
- ★ DYNASMART

- ★ <http://www.dynasmart.umd.edu/>



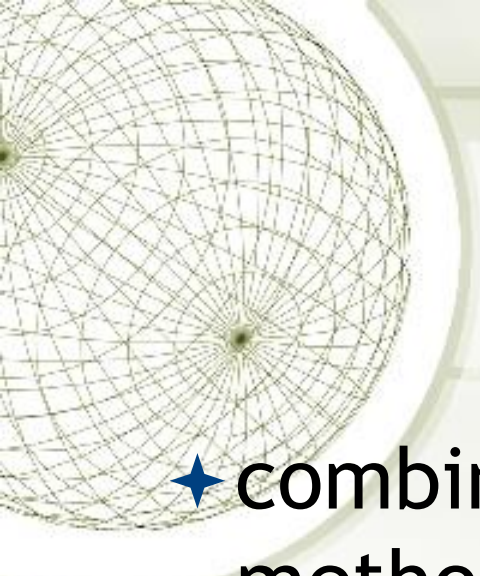
Mesosopic Details

- ★ Cell transmission
- ★ Hard to come by definite details
- ★ Traffic network is discretized
 - ★ Vehicles enter and leave discretization units on a schedule determined by:
 - ★ The road structure inside
 - ★ The number of cars inside
 - ★ The velocity of vehicles entering
 - ★ Units might be:
 - ★ One for each street & one for each intersection
 - ★ One for each metro area & one for each interstate



Mesosopic Details

- ★ Approaches a discrete microscale simulation when rules are simple and units are small.
- ★ Approaches a macroscale simulation as the units become larger and the rules more complex.



Hybrid Simulations

- ★ combine micro- and meso-scale methods
- ★ Modeling KY traffic
 - ★ Micro-scale for Louisville, Lexington, Northern Kentucky
 - ★ Meso-scale for interstates and major highways elsewhere



Concluding Remarks

- ★ Traffic simulation has been around for a long time.
 - ★ First known citation: 1955
- ★ Still active area.



References

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