

Collision Avoidance Analysis for Lane Changing and Merging¹

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Abstract

One of the riskiest maneuvers that a driver has to perform in a conventional highway system is to merge into the traffic and/or to perform a lane changing maneuver. Lane changing/merging collisions are responsible for one-tenth of all crash-caused traffic delays often resulting in congestion. Traffic delays and congestion, in general, increases travel time and has a negative economic impact.

In this paper, we analyze the kinematics of the vehicles involved in a lane changing/merging maneuver, and study the conditions under which lane changing/merging crashes can be avoided. That is, given a particular lane change/merge scenario, we calculate the minimum longitudinal spacing which the vehicles involved should initially have so that no collision, of any type, takes place during the maneuver.

Simulations of a number of examples of lane changing maneuvers are used to demonstrate the results. The results of this paper could be used to assess the safety of lane changing maneuvers and provide warnings or take evasive actions to avoid collision when combined with appropriate hardware on board of vehicles.

Keywords: Lane Changing, Lane Merging, Crash Avoidance, Minimum Safety Spacing (MSS).

1. Introduction

The inter-vehicle spacing or headway affects both safety and highway capacity. For collision free

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vehicle following, the spacing should be large enough in order to guarantee no collisions during all possible vehicle maneuvers. Lane changing/merging accidents consist of various types of vehicle collisions, such as rear-end collisions, single vehicle road departure accidents, side-wipe, and angle collisions. Lane changing/merging collisions constituted about 4.0% of all police-reported collisions in 1991, and accounted for about 0.5% of all fatalities [1].

Although the lane change crash problem is small relative to other types of crashes and does not account for a high percentage of traffic fatalities, this crash type is responsible for one-tenth of all crash-caused traffic delays often resulting into congestion. Traffic delays and congestion in general increases travel time and has a negative economic impact [2]. In practice, the possibility of merging collisions can be reduced by adjusting relative velocities and increasing the longitudinal inter-vehicles' spacing. Since roadway capacity is proportional to vehicle speed and inversely proportional to longitudinal inter-vehicle spacing, a large reduction in speed or a large increase in spacing leads to a low capacity highway system. For a high capacity highway system, the headway setting should be as small as possible. Since safety cannot be easily traded off, the choice of Minimum Safety Spacing (MSS) between vehicles for a collision free environment is important both from safety and capacity point of view.

Godbole, et. al. [3], and Shiller and Sundar [4] analyzed the obstacle avoidance problem. They considered the problem of collision between a moving vehicle and an existing static obstacle, such as a disabled vehicle or a large object. For avoiding the obstacle, two approaches were considered: stopping in the same lane, or performing a lane changing maneuver. Shiller and Sunder [4] developed what they called clearance and stopping curves that specify the collision avoidance maneuver. These curves divide the phase-plane (initial longitudinal velocity of vehicle under consideration versus its distance to the obstacle plane) into three regions. In region I the vehicle performs normal (non-emergency) full stop or normal lane changing maneuver. In region II the vehicle performs normal lane changing maneuver only, and in region III the vehicle performs full stop maneuver in the same lane. When the vehicle is in Region I, lane changing has priority over full stop if the lane changing is feasible. The vehicle is in Region III when a collision is imminent. In that case it is shown that an emergency stop in the same lane will reduce the severity of collision and is therefore preferable compared to lane changing. Godbole, et. al. [3] formulated the lane changing maneuver for obstacle avoidance problem as an optimal control problem. For normal lane changing the collision avoidance problem is formulated as an optimization problem where the time for completing the lane changing maneuver is minimized by selecting the appropriate lateral and longitudinal control inputs. In an emergency lane changing situation the lateral and longitudinal control inputs are calculated to minimize the longitudinal distance between the vehicle and the obstacle. Both aforementioned works, [3] and [4], do not take into account the vehicles in neighboring lanes, which could be obstacles to lane changing.

Bascunna [5] determined the conditions for safe and unsafe lane changing. The conditions were obtained for two vehicles involved in the lane changing maneuver. Initially the vehicles are travelling in two neighboring lanes and vehicle 1 performs a merging maneuver to the other lane. He considered four cases. Case I: The initial longitudinal velocity of vehicle 1 is less than that of vehicle 2, and vehicle 1 intends to complete the lane changing with constant longitudinal velocity, and then follows vehicle 2. Case II: The initial longitudinal velocity of vehicle 1 is less

than that of vehicle 2, and vehicle 1 intends to complete the lane changing by applying constant longitudinal acceleration and leads vehicle 2. Case III: The initial longitudinal velocity of vehicle 1 is greater than that of vehicle 2, and vehicle 1 intends to complete the lane changing with constant longitudinal velocity and leads vehicle 2. Case IV: The initial longitudinal velocity of vehicle 1 is greater than vehicle 2, and vehicle 1 intends to complete the lane changing with constant longitudinal deceleration and then follows vehicle 2.

In this paper, we examine the problem of safe lane changing and merging maneuvers in highway systems. By analyzing the kinematics of the vehicles involved in a lane changing or merging scenario, we present a general algorithm to calculate whether a particular lane changing/merging maneuver is safe, i.e., free of collisions. We present a general algorithm for calculating the Minimum Longitudinal Safety Spacing (MSS) for all vehicles involved. That is, given a particular lane change/merging scenario, we calculate the minimum longitudinal spacings that the vehicles, which affected by the lane changing maneuver, should initially follow for no collision to occur.

We examine special cases of lane changing/merging scenarios. For the cases where the merging vehicle moves with either constant longitudinal velocity [5] or acceleration, we explicitly calculate the MSSs and we show that the regions in the initial longitudinal spacing/ relative longitudinal velocity plane can be divided into safe and unsafe regions; once the vehicles start the lane changing/merging maneuver within the safe regions then collision-free maneuver is guaranteed. We finally analyze the switching longitudinal acceleration case [6], i.e., the case where the merging vehicle initially accelerates/decelerates with constant longitudinal acceleration in order to create enough spacing for the lane changing/merging maneuver and then it switches to another constant longitudinal acceleration/deceleration in order to adjust its velocity with the velocity in the destination lane. For this case we show that the results obtained for the constant acceleration case can be used in order to decide whether a particular lane changing/merging scenario is collision-free or not. In particular, we use the method of isoclines[7] to illustrate graphically the trajectory of the required lane changing maneuver that could move the vehicle from the unsafe region in to the safe one.

2. Minimum Longitudinal Safety Spacing

Let us consider a lane changing situation where vehicle M in Figure 1 will move from its current position between vehicles L_o and F_o to a new position between vehicles L_d and F_d in the neighboring lane. We refer to vehicles L_d , F_d , L_o , F_o and M as the leading vehicle in the destination lane, following vehicle in the destination lane, leading vehicle in the originating lane, following vehicle in the originating lane, and the vehicle which must perform the lane-changing (which will be called thereafter the merging vehicle), respectively.

Without loss of generality, we assume that the merging vehicle, M , starts the lane-changing maneuver at $t=0$. This maneuver consists of two parts. Initially M adjusts its longitudinal velocity and spacing for a time-interval t_{adj} , and then applies lateral acceleration to merge to the destination lane. In other words, t_{adj} is the required time for the merging vehicle to adjust its

longitudinal position and velocity before it starts merging to the destination lane.

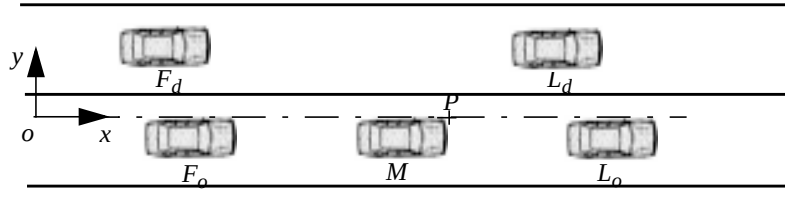


Fig 1: Pre-lane changing configuration showing position of merging vehicle M .

To measure the lateral and longitudinal positions of the vehicles involved in the maneuver, an arbitrary origin, which is denoted by "O" in Figure 1, is selected. The axis y is directed to ward the destination lane and the axis x is aligned with the lateral side of the merging vehicle which is closer to the destination lane. Figure 1 shows the top view of the pre-lane changing configuration of the vehicles involved in the lane changing maneuver where the destination lane is assumed to be on the left of the originating lane; axes x and y is chosen correspondingly. The origin "O" and the axes x, y in Figure 1 are assumed to be fixed at the beginning of the lane changing maneuver, $t=0$, till the end of maneuver.

Hereafter, the longitudinal acceleration/deceleration, the longitudinal velocity, the longitudinal position, and the lateral position of vehicle i will be denoted by $a_i(t)$, $v_i(t)$, $x_i(t)$, and $y_i(t)$, respectively, where $i \in \{L_d, F_d, L_o, F_o, M\}$. More precisely, $x_i(t)$ and $y_i(t)$ are, respectively, the longitudinal and lateral distances between the front-left corner of the vehicle i (e.g. denoted by "P" for merging vehicle in Fig. 1) and the origin "O".

With the exception of the merging vehicle, the lateral acceleration of all other vehicles is assumed to be zero. In our analysis, we assume a simple but realistic model for the lateral acceleration $a_{lat}(t)$ of the merging vehicle that is used to complete the lane change maneuver. It can be modeled as a Sine function of time [8], and is given as follows:

$$a_{lat}(t) = \begin{cases} \frac{2\pi H}{t_{lat}} \times \sin\left(\frac{2\pi}{t_{lat}}(t - t_{adj})\right) & t_{adj} \leq t \leq t_{lat} + t_{adj} \\ 0 & \text{Otherwise} \end{cases} \quad (1)$$

In equation 1, H is the total lateral displacement for the merging vehicle, t_{adj} is the time elapse before lateral acceleration applies, and t_{lat} is the total time, after t_{adj} , needed to complete the lane change. It should be noted that the lateral acceleration $a_{lat}(t)$, according to equation(1), is positive within the first half of the lateral displacement, i.e. $t < (t_{lat}/2) + t_{adj}$, and negative in the second half. Given $a_{lat}(t)$, the lateral velocity $v_{lat}(t)$, and lateral position $y_{lat}(t)$, of the front-left corner of the merging vehicle M (denoted by "p" in Figure 1) can be computed, easily.

A "lane change crash" occurs when the merging vehicle M attempts to change its lane and strikes or is struck by a vehicle in the adjacent lane. The model (1) is considered as an accurate model

for many simple lane change/merge maneuvers during which more than two-thirds of lane change/merge crashes occur [2].

The objective of this section is to use the simple lane change model described above and the longitudinal acceleration profiles of the five vehicles in Figure 1 to find the initial minimum longitudinal spacing between M and each of the other vehicles such that during a specified time-interval $[0, T]$, no collision, of any type, occurs. The length of the time interval T denotes the time under consideration. In the following subsections we develop analytical expressions for the minimum longitudinal spacing between the merging vehicle M and all other vehicles involved, in order to guarantee a lane change without collision.

2.1. Minimum Longitudinal Safety Spacing between M and L_d

Let's consider the vehicles M , merging vehicle, and L_d , leading vehicle in destination lane, during a lane change/merge maneuver as shown in figure 2. The merging vehicle, M , starts the lane-changing maneuver at $t=0$ by adjusting its longitudinal position and velocity, and then applying lateral acceleration at $t=t_{adj}$ according to (1). The type of collision between M and L_d could be of angle, side-wipe, or rear-end collision.

Let S denote the initial lateral distance between the upper side of the merging vehicle and the lower side of the vehicle L_d . Since the leading vehicle L_d remains in the destination lane, an angle and/or a side-wipe collision may occur as M passes the line LS in figure 2; LS is the tangent to the lower side of the leading vehicle L_d . The front-left corner of M is the first point of the merging vehicle which passes the line LS at the point C .

It should be noted that, since the lateral acceleration of the leading vehicle L_d is zero, the lateral position of L_d , y_{Ld} is constant.

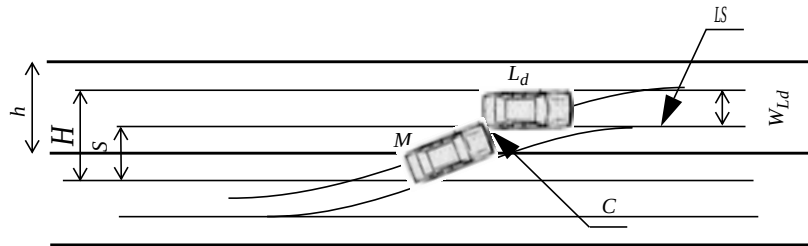


Fig 2: The marginal collision point between the merging vehicle M and the leading vehicle L_d .

Let $t_C + t_{adj}$ be the time-instant at which the front-left corner of the merging vehicle is at the point C in figure 2. The type of collision which may take place at or after this time-instant is angle, side-wipe or rear-end collision. An angle or a side-wipe collision may occur during or after the front-left corner of M passes the point C , i.e., at or after the time-instant $t_C + t_{adj}$. A rear-end collision may occur after the vehicle M accomplishes the lane changing maneuver, i.e., after the merging vehicle has completely merged. The time-instant $t_C + t_{adj}$ can be found by solving the

following equation for $t=t_C+t_{adj}$,

$$y_{lat}(t) = S = y_{Ld} - w_{Ld} \quad (2)$$

where w_{Ld} is the width of the leading vehicle L_d . By taking all types of collisions mentioned above into account, the condition for no collision between M and L_d is given by:

$$x_M(t) < x_{Ld}(t) - l_{Ld} - w_M \times \sin(\theta(t)) \quad \forall t \in [t_C + t_{adj}, T] \quad (3)$$

where l_{Ld} is the length of the leading vehicle L_d , w_M is the width of the merging vehicle M , and $\theta(t)$ is the angle between the tangent of the lane changing trajectory at the point $y_{lat}(t)$ and the horizontal axis. The last term in inequality (3), $w_M \times \sin(\theta(t))$, is to prevent any angle collision between any point on the front bumper of the merging vehicle M and the rear-right corner of leading vehicle L_d in the time interval $[t_C+t_{adj}, t_{lat}+t_{adj}]$. From the definition of θ , it follows:

$$\tan(\theta(t)) = \frac{\partial y_{lat}(t)}{\partial x_M(t)} = \frac{\partial y_{lat}(t)/\partial t}{\partial x_M(t)/\partial t} = \frac{v_{lat}(t)}{v_M(t)} \quad (4)$$

Equation (4) indicates that the value of $\tan(\theta(t))$ and consequently $\sin(\theta(t))$ can be evaluated at each time instant based on the lateral and longitudinal velocity of the merging vehicle. The maximum value of $\theta(t)$ and consequently the maximum value of $\sin(\theta(t))$ in inequality(3) will be at the time instant $t=t_C+t_{adj}$. (Note that $t_C+t_{adj} > (t_{lat}/2)+t_{adj}$, i.e. $a_{lat}(t_C+t_{adj}) < 0$). Let's define

$$l_{L1} = l_{Ld} + w_M \times \underset{t}{\text{Max}}(\sin\theta(t)) \equiv l_{Ld} + w_M \times \sin(\theta(t_C + t_{adj})), \text{ then inequality (3) can be}$$

simplified as follows:

$$x_M(t) < x_{Ld}(t) - l_{L1} \quad \forall t \in [t_C + t_{adj}, T] \quad (5)$$

Let $Sr(t)$ be the longitudinal spacing between point P of vehicle M and the rear end (bumper) of vehicle L_d (Note that $y_{Ld}(t)=0$). That is:

$$Sr(t) = x_{Ld}(t) - l_{L1} - x_M(t) \quad \forall t \in [t_C + t_{adj}, T] \quad (6)$$

As long as the longitudinal spacing is greater than zero, i.e. $Sr(t) > 0$ for $t \geq t_C+t_{adj}$, no collision will occur during the lane-changing maneuver. Based on (5), we can rewrite equation (6) as:

$$Sr(t) = \left(Sr(0) + \int_0^t \int_0^\lambda (a_{Ld}(\tau) - a_M(\tau)) d\tau d\lambda + (v_{Ld}(0) - v_M(0))t \right) > 0 \quad (7)$$

$$\forall t \in [t_C + t_{adj}, T]$$

where $Sr(0) = x_{Ld}(0) - l_{Ld} - x_M(0)$. Our objective is to find the initial minimum value of $Sr(0)$ which guarantees no collision between the leading vehicle L_d and the merging vehicle M . The minimum value of $Sr(0)$ is the minimum initial longitudinal relative spacing between L_d and M , for collision free vehicle merging and is denoted by $MSS(L_d, M)$. It is calculated using (7) as follows,

$$MSS(L_d, M) = \underset{t}{Max} \left(\int_0^t \int_0^\lambda (a_M(\tau) - a_{L_d}(\tau)) d\tau d\lambda + (v_M(0) - v_{L_d}(0))t \right) \quad (8)$$

$$\forall t \in [t_C + t_{adj}, T]$$

From equation (8) it is clear that the minimum initial longitudinal relative spacing between L_d and M , $MSS(L_d, M)$, depends on the relative longitudinal acceleration, the relative initial longitudinal velocity between the two vehicles, as well as the time interval $[t_C + t_{adj}, T]$. This time interval depends on the lateral distance S , the lateral time t_{lat} , and the adjustment time t_{adj} .

2.2. Minimum Longitudinal Safety Spacing between M and F_d :

Now, let us consider the merging vehicle, M , and the following vehicle in the destination lane, F_d , during a lane changing/merging maneuver as shown in figure 3. The merging vehicle, M , starts its merging maneuver at $t=0$, and its lateral movement at $t=t_{adj}$. The type of possible collision between M and F_d could be of angle, side-wipe, and rear-end collision.

Since the following vehicle F_d is in the destination lane and has zero lateral motion, an angle and/or a side-wipe collision may occur during or after the vehicle M passes the line LS in figure 3, where the line LS is the tangent to the lower side of the following vehicle F_d . We define the point C as the intersection between the rear-left corner of the vehicle M and the line LS; obviously the point C is the marginal point that a collision between the two vehicles could occur.

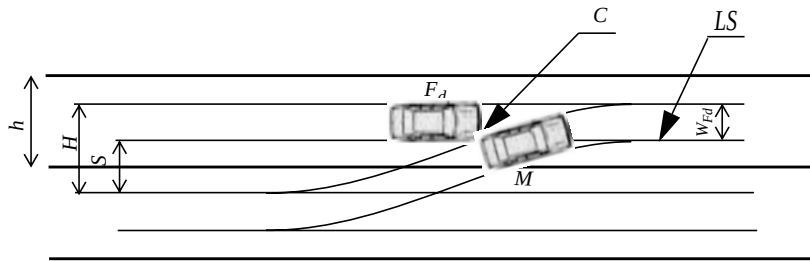


Fig 3: The marginal collision point between the merging vehicle M and the following vehicle F_d .

In this case we need to find the coordinates of the point C as well as the time-instant at which the rear-left corner of the merging vehicle passes this point. In order to calculate the motion of the rear-left corner of the merging vehicle, we apply first order approximation (tangent to the vehicle's path) to calculate the lateral position of the other corners of the merging vehicle as

follows:

$$\begin{aligned}
y_{rear-left}(t) &\cong y_{lat}(t) - l_M \times \sin(\theta(t)) \\
y_{front-right}(t) &\cong y_{lat}(t) - w_M \times \cos(\theta(t)) \\
y_{rear-right}(t) &\cong y_{lat}(t) - (l_M \times \sin(\theta(t)) + w_M \times \cos(\theta(t)))
\end{aligned} \tag{9}$$

where $\theta(t)$ is the angle between the tangent of the path at point $y_{lat}(t)$ and the horizontal axis, and l_M , w_M are the length and width of the merging vehicle, respectively.

Let $t_C + t_{adj}$ be the time-instant at which the rear-left corner of the merging vehicle is at point C in Figure 3. Using equation (9), the time-instant $t_C + t_{adj}$ can be found by solving the following equation:

$$y_{lat}(t) - l_M \times \sin(\theta(t)) \cong S = y_{Fd} - w_{Fd} \tag{10}$$

Where w_{Fd} is the width of the following vehicle F_d . Using equation (4), equation (10) can be rewritten as,

$$y_{lat}(t) - l_M \times \frac{v_{lat}(t)}{\sqrt{v_{lat}^2(t) + v_M^2(t)}} \cong S \tag{11}$$

Considering all possible types of collision, the condition for collision avoidance between M and F_d would be:

$$x_{Fd}(t) < x_M(t) - l_M \times \cos(\theta(t)) \quad \forall t \in [t_C + t_{adj}, T] \tag{12}$$

It should be noted that the maximum value of $\cos(\theta(t))$, in the time interval $[t_C + t_{adj}, T]$, will be at or after the time instant $t = t_{lat} + t_{adj}$, where the value of $\theta(t)$ is minimum and is equal to zero. Therefore, equation (12) can be simplified as follows:

$$x_{Fd}(t) < x_M(t) - l_M \quad \forall t \in [t_C + t_{adj}, T] \tag{13}$$

The above approximation results is a conservative condition for no collision condition during the time interval $t_C + t_{adj} \leq t \leq t_{lat} + t_{adj}$, i.e. before M completes its lane changing maneuver. The longitudinal spacing between the rear of the vehicle M and the front of the vehicle F_d is given by:

$$Sr(t) = x_M(t) - l_M - x_{Fd}(t) \quad \forall t \in [t_C + t_{adj}, T] \tag{14}$$

As long as the longitudinal spacing in (14) is positive, i.e. $Sr(t) > 0$ for $t \geq t_C + t_{adj}$, no collision

occurs. Based on (13), we can rewrite equation (14) as follows:

$$Sr(t) = \left(Sr(0) + \int_0^t \int_0^\lambda (a_M(\tau) - a_{Fd}(\tau)) d\tau d\lambda + (v_M(0) - v_{Fd}(0))t \right) > 0 \quad (15)$$

$$\forall t \in [t_C + t_{adj}, T]$$

where $Sr(0) = x_M(0) - l_M - x_{Fd}(0)$. is the initial longitudinal relative spacing between F_d and M . For collision free vehicle merging, the minimum value of $Sr(0)$ is denoted by $MSS(M, F_d)$ and is calculated using (15) as follows:

$$MSS(M, F_d) = \underset{t}{Max} \left(\int_0^t \int_0^\lambda (a_{Fd}(\tau) - a_M(\tau)) d\tau d\lambda + (v_{Fd}(0) - v_M(0))t \right) \quad (16)$$

$$\forall t \in [t_C + t_{adj}, T]$$

From equation (16) it is clear that the minimum initial longitudinal relative spacing between F_d and M , $MSS(M, F_d)$, depends on the relative longitudinal acceleration, the relative initial longitudinal velocity between the two vehicles, as well as the time interval $[t_C + t_{adj}, T]$.

2.3. Minimum Longitudinal Safety Spacing between M and L_o :

Consider now the case of the merging vehicle, M , and the leading vehicle in the originating lane, L_o , during the lane changing maneuver shown in figure 4. The vehicle M starts its merging maneuver at $t=0$, and its lateral movement at $t=t_{adj}$. The type of collision between M and L_o could be of angle, or rear-end collision, but not side-wipe collision.

Since the leading vehicle L_o remains in the originating lane, an angle collision may occur before the front-right corner of M passes the line LS at point C in figure 4.

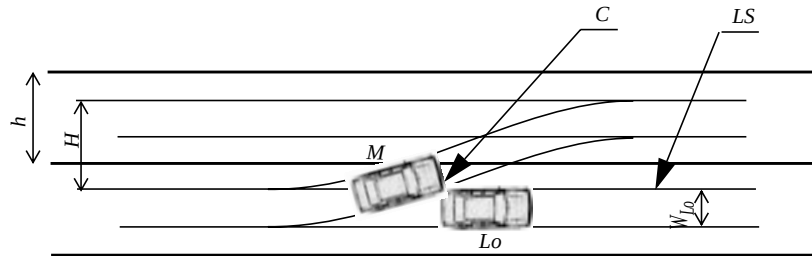


Fig 4: The marginal collision point between the merging vehicle M and the leading vehicle L_o .

Since the lateral acceleration of the leading vehicle L_o is zero, the lateral position of L_o , y_{L_o} , is constant.

Let $t_C + t_{adj}$ be the time-instant at which the front-right corner of the merging vehicle is at the point C as shown in figure 4. Considering the first order approximation in (9), the time-instant $t_C + t_{adj}$ can be found by solving the following equation.

$$y_{lat}(t) - w_M \times \cos(\theta(t)) \cong S = y_{Lo} \quad (17)$$

where w_M is the width of the merging vehicle M . Using (4), equation (17) can be rewritten as:

$$y_{lat}(t) - w_M \times \frac{v_M(t)}{\sqrt{v_{lat}^2(t) + v_M^2(t)}} \cong S \quad (18)$$

Considering all types of collision, the condition for collision avoidance between M and L_o is:

$$x_M(t) < x_{Lo}(t) - l_{Lo} - w_M \times \sin(\theta(t)) \quad \forall t \in [0, t_C + t_{adj}] \quad (19)$$

where l_{Lo} is the length of the leading vehicle L_o , and $\theta(t)$ is the angle between the tangent of the path at point $y_{lat}(t)$ and the horizontal axis.

The last term in equation (19), $w_M \times \sin(\theta(t))$, is to prevent any angle collision between any point on the rear bumper of the leading vehicle L_o and the front-right corner of the merging vehicle M in the time interval $[t_{adj}, t_C + t_{adj}]$. It should be noted that the maximum value of $\theta(t)$ and consequently the maximum value of $\sin(\theta(t))$ in equation (19) will be at the time instant $t = t_C + t_{adj}$. Let's define $l_{L1} = l_{Lo} + \max(\sin(\theta(t)))$, then equation (19) can be rewritten as follows:

$$x_M(t) < x_{Lo}(t) - l_{L1} \quad \forall t \in [0, t_C + t_{adj}] \quad (20)$$

According to (20), the longitudinal spacing between the front of the vehicle M and the rear of vehicle L_o would be:

$$Sr(t) = x_{Lo}(t) - l_{L1} - x_M(t) \quad \forall t \in [0, t_C + t_{adj}] \quad (21)$$

As long as the spacing $Sr(t)$ is greater than zero, i.e. $Sr(t) > 0$ for $t \leq t_C + t_{adj}$, no collision occurs. According to (20), we can rewrite equation (21) as:

$$Sr(t) = \left(Sr(0) + \int_0^t \int_0^\lambda (a_{Lo}(\tau) - a_M(\tau)) d\tau d\lambda + (v_{Lo}(0) - v_M(0))t \right) > 0 \quad (22)$$

$$\forall t \in [0, t_C + t_{adj}]$$

The minimum value of $Sr(0)$ is the minimum initial longitudinal relative spacing between L_o and M , for collision free vehicle merging and is denoted by $MSS(L_o, M)$.

$$MSS(L_o, M) = \max_t \left\{ \left(\int_0^t \int_0^\lambda (a_M(\tau) - a_{L_o}(\tau)) d\tau d\lambda + (v_M(0) - v_{L_o}(0))t \right), 0 \right\} \quad (23)$$

$$\forall t \in [0, t_C + t_{adj}]$$

Equation (23) indicates that $MSS(L_o, M)$ depends on the relative longitudinal acceleration, the relative initial velocity between the two vehicles, as well as the time interval $[0, t_C + t_{adj}]$.

2.4. Minimum Longitudinal Safety Spacing between M and F_o :

In this subsection, we consider the merging vehicle M , and the following vehicle F_o in the originating lane, during a lane change/merge maneuver as shown in figure 5. The merging vehicle, M , starts its merging maneuver at $t=0$, and its lateral movement at $t=t_{adj}$. The types of possible collision between M and F_o could be of angle, and rear-end collision, but not side-wipe collision.

Since the following vehicle F_o is in the originating lane and has zero lateral motion, an angle and/or a rear-end collision may occur during or before the vehicle M passes the line LS in figure 5, where LS is the tangent to the upper side of the following vehicle F_o .

We define the point C as the intersection between the rear-right corner of the vehicle M and the line LS ; obviously the point C is the marginal point that a collision between two vehicles may occur.

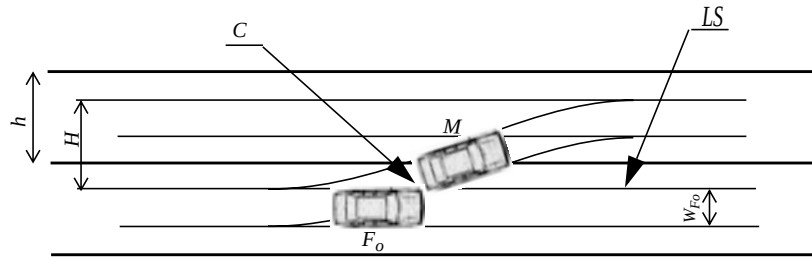


Fig 5: The last possible collision point between the merging vehicle M and the following vehicle F_o .

Since the following vehicle in originating lane, F_o , has no lateral acceleration, the lateral position of y_{F_o} is constant. Let $t_C + t_{adj}$ be the time-instant at which the rear-right corner of the merging vehicle is at the point C in figure 5. Using the first order approximation in (9), the time-instant $t_C + t_{adj}$ can be found by solving the following equation.

$$y_{lat}(t) - (l_M \times \sin(\theta(t)) + w_M \times \cos(\theta(t))) \cong S = y_{F_o} \quad (24)$$

Using (4), equation (24) can be rewritten as follows:

$$v_{lat}(t) - \left(l_M \times \frac{v_{lat}(t)}{\sqrt{v_{lat}^2(t) + v_M^2(t)}} + w_M \times \frac{v_M(t)}{\sqrt{v_{lat}^2(t) + v_M^2(t)}} \right) \cong S \quad (25)$$

Considering all possible types of collision, the condition for no collision between M and F_o is given by:

$$x_{Fo}(t) < x_M(t) - l_M \times \cos(\theta(t)) \quad \forall t \in [0, t_C + t_{adj}] \quad (26)$$

It should be noted that the maximum value of $\cos(\theta(t))$, in the time interval $[0, t_C + t_{adj}]$, will be before the time instant $t = t_{adj}$, where the value of $\theta(t)$ is minimum and is equal to zero. Therefore, equation (26) can be simplified as follows:

$$x_{Fo}(t) < x_M(t) - l_M \quad \forall t \in [0, t_C + t_{adj}] \quad (27)$$

Equation(27) results in a conservative condition for no collision during the time interval $[t_{adj}, t_C + t_{adj}]$, i.e. after M starts its lane changing maneuver. According to (27), the longitudinal spacing between the rear of the vehicle M and the front of the vehicle F_o is given by:

$$Sr(t) = x_M(t) - l_M - x_{Fo}(t) \quad \forall t \in [0, t_C + t_{adj}] \quad (28)$$

As long as the longitudinal spacing in (28) is greater than zero, i.e. $Sr(t) > 0$ for $t \leq t_C + t_{adj}$, no collision occurs. Based on (27), we can rewrite equation (28) as follows:

$$Sr(t) = \left(Sr(0) + \int_0^t \int_0^\lambda (a_M(\tau) - a_{Fo}(\tau)) d\tau d\lambda + (v_M(0) - v_{Fo}(0))t \right) > 0 \quad (29)$$

$$\forall t \in [0, t_C + t_{adj}]$$

The minimum value of $Sr(0)$ is the minimum initial longitudinal relative spacing between M and F_o , for collision free vehicle merging and is denoted by $MSS(M, Fo)$. It is calculated using (29) as follows:

$$MSS(M, Fo) = \max_t \left\{ \left(\int_0^t \int_0^\lambda (a_{Fo}(\tau) - a_M(\tau)) d\tau d\lambda + (v_{Fo}(0) - v_M(0))t \right), 0 \right\} \quad (30)$$

$$\forall t \in [0, t_C + t_{adj}]$$

From equation (30) it is clear that $MSS(M, Fo)$ depends on the relative longitudinal acceleration, the relative initial longitudinal velocity between two vehicles, as well as the time interval $t_C + t_{adj}$.

This time interval depends on the lateral position y_{Fo} , lateral time t_{lat} , and adjustment t_{adj} .

In all the expressions for MSSs, the value of MSS depends on the value of t_{adj} , the time elapsed before lateral acceleration is applied. During this time the merging vehicle M adjusts its longitudinal position and velocity before it starts merging.

3. Special Cases and Simulations

In this section we consider specific profiles for the longitudinal acceleration of the vehicles involved in lane changing maneuvers in order to derive closed form expressions for MSS. We assume constant longitudinal velocity for all vehicles in Figure 1 except for M whose longitudinal velocity may change when the lane changing/merging scenario starts. In the following subsections we calculate the minimum longitudinal safety spacing MSS as a function of the relative longitudinal velocity between the merging vehicle M and each of the other vehicles in figure 1 by considering two cases:

Case I: The merging vehicle M performs the merging scenario with constant longitudinal velocity. Obviously, the longitudinal velocity of the merging vehicle will remain the same as the longitudinal velocity of the vehicles in the originating lane, i.e. the velocity before starting its maneuver. Case II: The merging vehicle M applies a constant longitudinal acceleration/deceleration in order to reach the longitudinal velocity of the vehicles in the destination lane after the specific time $t_{long} + t_{adj}$.

In each case, and based on the relative longitudinal velocity and position between M and the other vehicles, the merging vehicle M can determine whether the merging scenario is safe or unsafe before the merging maneuver starts.

3.1. Constant Longitudinal Velocity

This is the case where all five vehicles are moving with constant longitudinal velocity, i.e. the longitudinal acceleration for all vehicles is zero, $a_i(t)=0$ for $\forall t \in [0, T]$ and $i \in \{L_d, F_d, L_o, F_o, M\}$. The merging vehicle M keeps its longitudinal velocity constant throughout the merging maneuver.

Minimum Longitudinal Safety Spacing between M and L_d

According to (7), the condition for collision avoidance between M and L_d with constant longitudinal velocity is:

$$Sr(t) = (Sr(0) + (v_{Ld} - v_M)t) > 0 \quad \forall t \in [t_C + t_{adj}, T] \quad (31)$$

Since the relative longitudinal velocity, $v_M - v_{Ld}$, is constant, the minimum initial longitudinal safety spacing, $MSS(Ld, M)$ is:

$$MSS(Ld, M) = \begin{cases} (v_M - v_{Ld}) \times T & v_M - v_{Ld} \geq 0 \\ (v_M - v_{Ld}) \times (t_C + t_{adj}) & \text{Otherwise} \end{cases} \quad (32)$$

Minimum Longitudinal Safety Spacing between M and F_d

Equation (15) provides the condition for collision avoidance between M and F_d. In the case of constant longitudinal velocity, this condition is as follows:

$$Sr(t) = (Sr(0) + (v_M - v_{Fd})t) > 0 \quad \forall t \in [t_C + t_{adj}, T] \quad (33)$$

And, the minimum initial longitudinal safety spacing, MSS(M, F_d) will be:

$$MSS(M, Fd) = \begin{cases} (v_{Fd} - v_M) \times T & v_{Fd} - v_M \geq 0 \\ (v_{Fd} - v_M) \times (t_C + t_{adj}) & \text{Otherwise} \end{cases} \quad (34)$$

Minimum Longitudinal Safety Spacing between M and L_o

According to (22), the condition for collision avoidance between M and L_o with constant longitudinal velocity is as follows:

$$Sr(t) = (Sr(0) + (v_{Lo} - v_M)t) > 0 \quad \forall t \in [0, t_C + t_{adj}] \quad (35)$$

The minimum initial longitudinal safety spacing, MSS(L_o, M) will be:

$$MSS(Lo, M) = \begin{cases} (v_M - v_{Lo}) \times (t_C + t_{adj}) & v_M - v_{Lo} \geq 0 \\ 0 & \text{Otherwise} \end{cases} \quad (36)$$

Minimum Longitudinal Safety Spacing between M and F_o

According to equation (29), the condition for collision avoidance between M and F_o with constant longitudinal velocity is as follows:

$$Sr(t) = (Sr(0) + (v_M - v_{Fo})t) > 0 \quad \forall t \in [0, t_C + t_{adj}] \quad (37)$$

The minimum initial longitudinal safety spacing, MSS(M, F_o) will be:

$$MSS(M, Fo) = \begin{cases} (v_{Fo} - v_M) \times (t_C + t_{adj}) & v_{Fo} - v_M \geq 0 \\ 0 & \text{Otherwise} \end{cases} \quad (38)$$

Simulation results:

Simulations are used to demonstrate equations (32), (34), (36), and (38). In these simulations, we

set the time $T=50$ sec, the adjustment time $t_{adj}=0$ sec, the lateral time, in equation (1), $t_{lat}=5$ sec, and the lateral displacement $H=12$ feet. Figures 6, 7, 8, and 9 show the initial relative longitudinal spacings versus the relative longitudinal velocity between M and the other four vehicles involved in the lane-changing maneuver. The solid lines (which will be called thereafter *safety margins*) in these figures represent the margins between safe and unsafe lane changing regions.

For positive relative velocity, $v_M - v_{Ld}$, in figure 6, the safety margin is a line with slope equal to $T=50$. For negative relative velocity, it is approximately a line with slope equal to 2.8 which is the value of $t_C + t_{adj}$ in equation (2). Thus for constant longitudinal velocity, the safety margin consists of two lines passing through the origin with different tangents. This conclusion complies with equation (31).

A similar situation appears in figures 7 and 8 for the spacings between M and F_d and M and L_o , i.e. the safety margins consist of two lines with different tangents which are given by equations (33) and (35). In the case of the spacing between M and F_o in figure 9, one of the components of the safety margin is not a straight line; the reason for this is due to the approximation we made in equation (25). The smoother the lane changing trajectory is, the more accurate the approximation is in (25).

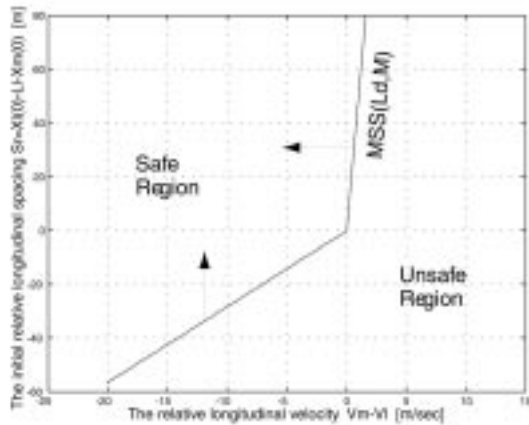


Fig 6: The collision region between M and L_d , Case I: Constant velocity.

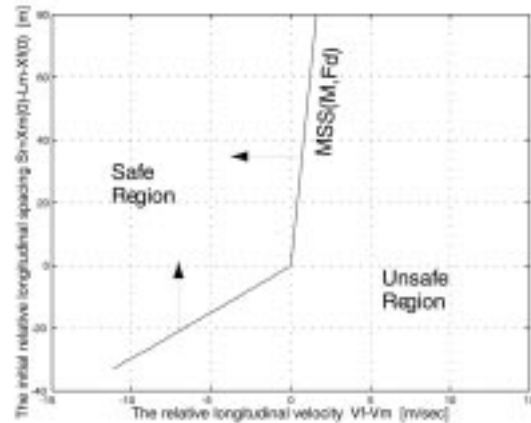


Fig 7: The collision region between M and F_d ; Case I: Constant velocity.

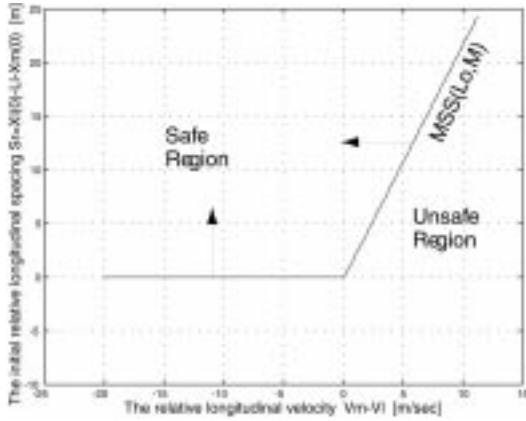


Fig 8: The collision region between M and L_d ; Case I: Constant velocity.

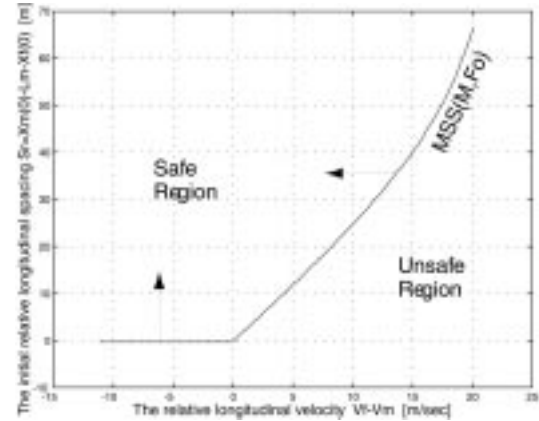


Fig 9: The collision region between M and F_0 ; Case I: Constant velocity.

3.2. Switching Longitudinal Acceleration

In this subsection we will examine the acceleration profile proposed in [6] for lane changing. According to [6] all vehicles in both lanes, except M , are moving with constant longitudinal velocity, i.e. $a_i(t)=0$ for $i \in \{L_d, F_d, L_o, F_o\}$, while the longitudinal acceleration profile of the merging vehicle, M , is the one plotted in Figure 10. More precisely, the merging vehicle initially accelerates/decelerates with constant longitudinal acceleration a_{adj} , in order to create enough spacing with the rest four vehicles in Figure 1. At the time-instant t_{adj} , the merging vehicle starts merging and it switches its longitudinal acceleration to a_M ; the merging vehicle continues to accelerate with acceleration a_M , until its longitudinal velocity becomes equal to the velocity of the vehicles in the destination lane at the time-instant $t_{adj}+t_{long}$. After this time-instant the merging vehicle's longitudinal acceleration becomes zero.

In the following discussions and in order to get some insight about the problem, we first analyze the case where $t_{adj}=0$ and then the more general case $t_{adj}>0$.

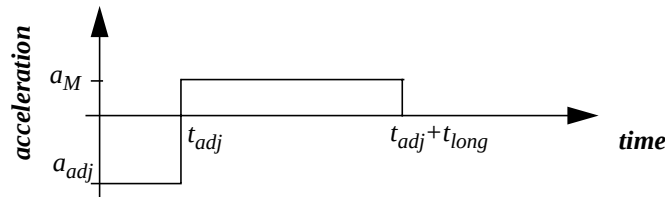


Fig 10: The switching longitudinal accelerating profile of merging vehicle, M .

Minimum Longitudinal Safety Spacing between M and L_d

1) $t_{adj}=0$

In this case, the velocity of all vehicles except M is constant while the velocity of the vehicle M become equal to the velocity of L_d at the time instant t_{long} and remain constant thereafter.

Therefore, the value of the longitudinal acceleration of the merging vehicle, a_M , is as follows:

$$a_M = \begin{cases} \frac{v_{Ld} - v_M(0)}{t_{long}} & t \leq t_{long} \\ 0 & \text{Otherwise} \end{cases} \quad (39)$$

According to equation (7) the condition for collision avoidance between M and L_d is:

$$Sr(t) = \left(Sr(0) + (v_{Ld} - v_M(0)) \times \left(t - \frac{t^2}{2t_{long}} \right) \right) > 0 \quad \forall t \in [t_C, t_{long}] \quad (40)$$

Considering different values of initial relative longitudinal velocity, $v_M(0) - v_{Ld}$, the minimum initial longitudinal safety spacing, $MSS(L_d, M)$ can be obtained analytically as follows:

$$MSS(L_d, M) = \begin{cases} (v_M(0) - v_{Ld}) \times t_{long} / 2 & v_M(0) - v_{Ld} \geq 0 \\ (v_M(0) - v_{Ld}) \times t_C & \text{Otherwise} \end{cases} \quad (41)$$

Simulations are used to demonstrate the result in equation (41). In these simulations, we set parameters for lane changing profile as those in section 3.1; and $t_{long} = 10$ sec. The safe and unsafe region is shown in figure 11.

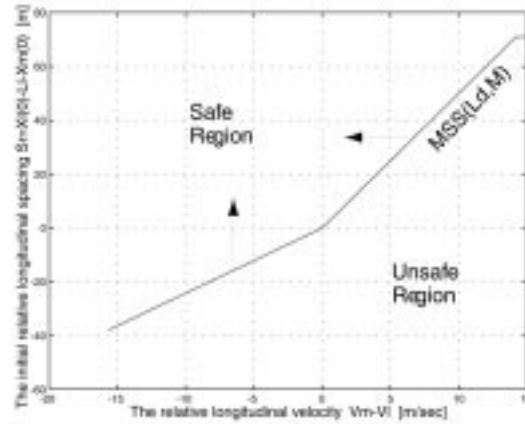


Fig 11: The collision region between M and L_d ; Case II: switching longitudinal acceleration.

Figure 11 shows the initial relative longitudinal distance versus the relative longitudinal velocity between M and L_d . The solid curve is the safety margin between safe and unsafe regions. For positive relative velocities, $v_M(0) - v_{Ld}$, the safety margin corresponds to a line with slope equal to $t_{long}/2 = 5$ while for negative relative velocities, it is a line with tangent equal to 2.8 which is the value of $t_C + t_{adj}$ in equation (2).

Comparison between figures 6 and 11 indicates that the safe region has been expanded. Therefore the switching longitudinal acceleration policy with $t_{adj}=0$ is more reliable than the constant longitudinal velocity policy for the case of vehicles M and L_d .

II) $t_{adj}>0$:

For the case of $t_{adj}=0$ even if initially the two vehicles' relative spacing and velocity is in the unsafe region, it may happen that no collision occurs due to the switching acceleration policy. The initial relative spacing and velocity between the vehicles M and L_d , defines a starting point in figure 11. If this point happens to be in the safe area, there is no need to apply any adjustment acceleration, a_{adj} to the merging vehicle, since the lane changing will be safe. If the initial relative spacing and velocity of the two vehicles belong to the unsafe region, then we need to apply the switching acceleration policy, in order to achieve appropriate relative spacing and velocity before the merging vehicle starts merging. In other words, we want to move from the unsafe region into the safe region and then start merging. Let's now define the following state space variables:

$$\begin{aligned} x_1 &= x_{Ld} - x_M - l_{L1} \\ x_2 &= v_M - v_{Ld} \end{aligned} \quad (42)$$

By differentiating the above variables with respect to time, it is easy to see that the following equations are valid,

$$\begin{aligned} \dot{x}_1 &= \dot{x}_{Ld} - \dot{x}_M = v_{Ld} - v_M = -x_2 \\ \dot{x}_2 &= \dot{v}_M = a_{adj} \end{aligned} \quad (43)$$

Using the technique of isoclines [7], and solving the differential equation in (43), we obtain the isoclines as follows:

$$x_1 = -\frac{x_2^2}{2a_{adj}} + c \quad (44)$$

The constant c is the integration constant which depends on the initial values, $x_1(0)$ and $x_2(0)$.

Figure 12 shows the isoclines that correspond to different values of a_{adj} for the simulation in figure 11. The initial state (initial relative spacing and velocity) has been chosen to be in the unsafe region. Applying negative a_{adj} , it is possible to move into the safe region in order to start the merging maneuver. The larger the absolute value of a_{adj} is, the faster we move into the safe region. The minimum value of t_{adj} for each a_{adj} is determined by the point of intersection between the corresponding isocline curve of a_{adj} and the safety margin in figure 12.

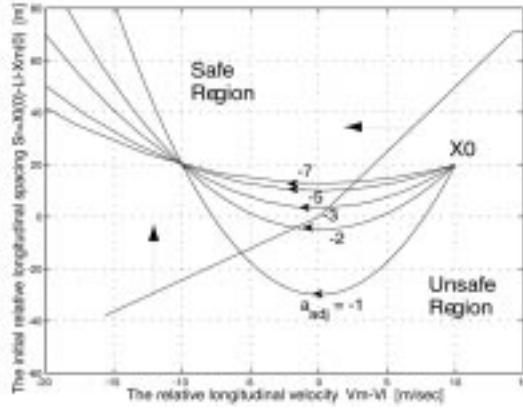


Fig 12: Applying a_{adj} to move from unsafe area into safe area.

It should be noted that, a_{adj} is limited by the acceleration/braking capabilities of the vehicle. Moreover, in order to maintain passenger comfort, the $|a_{adj}|$ must be less than a value say a_{comf} which defines the maximum acceleration that maintains passenger comfort [6]. In addition, large $|a_{adj}|$ results in larger "shock wave" propagating down the originating lane as well as larger minimum initial longitudinal relative spacing $S_r(0)$, which in turn reduces the throughput of the highway.

In the above discussion we assumed that we can apply a_{adj} for a period of t_{adj} seconds. This may not be always feasible because the vehicle's velocity may exceed its limits or, even worse, we may have the situation where the velocity of the vehicle becomes zero. In this case the acceleration profile of figure 10 cannot be applied; however, one can use the modified acceleration profile shown in figure 13.

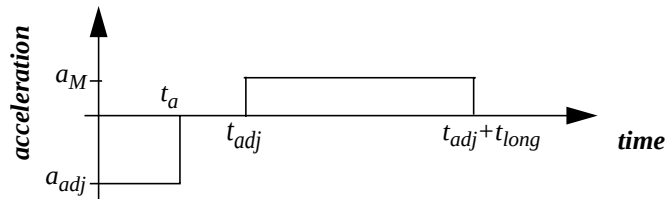


Fig 13: The modified switching longitudinal accelerating profile of merging vehicle, M .

Constant velocity in the interval $[t_a, t_{adj}]$ will help the merging vehicle to create enough relative spacing in order to enter the safe region. In this case, and for the time-interval $[t_a, t_{adj}]$, the state space equations can be rewritten as:

$$\begin{aligned}\dot{x}_1 &= -x_2 \\ \dot{x}_2 &= 0\end{aligned}\tag{45}$$

which results in,

$$\begin{aligned} x_2 &= c = x_2(t_a) \\ x_1 &= (-x_2(t_a))(t - t_a) + x_1(t_a) \quad \forall t \in [t_a, t_{adj}] \end{aligned} \quad (46)$$

If we intend to increase the relative spacing, x_1 , we have to make sure that $x_2(t_a)$ has a negative value (negative relative acceleration). In other words, we have to make sure that, at $t=t_a$, we are in the left half plane in figure 11 and then we can set a_{adj} equal to zero in order to move into the safe region. Figure 14 shows the trajectory of moving from the unsafe into the safe region.

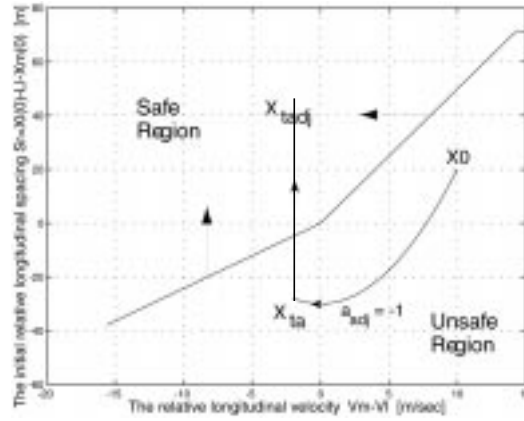


Fig 14: Applying modified a_{adj} to move from unsafe area into safe area.

Minimum Longitudinal Safety Spacing between M and F_d

1) $t_{adj}=0$:

Here, we assume that the longitudinal velocity of all vehicles except M is constant while the longitudinal velocity of M will be increasing/decreasing until it becomes equal to the velocity of F_d at $t=t_{long}$. Therefore the value of a_M will be:

$$a_M = \begin{cases} \frac{v_{Fd} - v_M(0)}{t_{long}} & t \leq t_{long} \\ 0 & \text{Otherwise} \end{cases} \quad (47)$$

According to equation (15) the condition for collision avoidance between M and F_d is:

$$Sr(t) = \left(Sr(0) + (v_M(0) - v_{Fd}) \times \left(t - \frac{t^2}{2t_{long}} \right) \right) > 0 \quad \forall t \in [t_C, t_{long}] \quad (48)$$

And the minimum initial longitudinal safety spacing, $MSS(M, F_d)$ is:

$$MSS(M, F_d) = \begin{cases} (v_{Fd} - v_M(0)) \times t_{long}/2 & v_{Fd} - v_M(0) \geq 0 \\ (v_{Fd} - v_M(0)) \times t_C & \text{Otherwise} \end{cases} \quad (49)$$

We use simulations to demonstrate the above results. The simulations parameters are set as those in previous subsection (safety spacing between M and L_d); The safe and unsafe regions are shown in figure 15. In figure 15 for initial positive relative velocities, $v_{Fd} - v_M(0)$, the safety margin is a line with slope equal to $t_{long}/2=5$. For negative relative velocities, it is a line with tangent equal to 2.95 which is the value of $t_C + t_{adj}$ in equation (10).

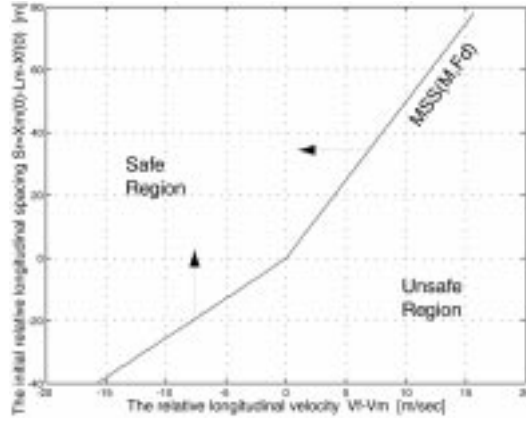


Fig 15: The collision region between F_d and M ; Case II: switching longitudinal acceleration.

Comparison between figures 7 and 15 indicates that the safe region has been expanded here, too.

II) $t_{adj} > 0$:

Similar to the case of vehicles F_d and M , we define the state variables as follows:

$$\begin{aligned} x_1 &= x_M - x_{Fd} - l_M \\ x_2 &= v_{Fd} - v_M \end{aligned} \quad (50)$$

and we differentiate them with respect to time, to obtain,

$$\begin{aligned} \dot{x}_1 &= \dot{x}_M - \dot{x}_{Fd} = v_M - v_{Fd} = -x_2 \\ \dot{x}_2 &= -\dot{v}_M = -a_{adj} \end{aligned} \quad (51)$$

Therefore the isoclines are as follows:

$$x_1 = \frac{x_2^2}{2a_{adj}} + c \quad (52)$$

Here c is a constant that depends on the initial values, $x_1(0)$ and $x_2(0)$. Figure 16 shows the

isoclines that correspond to different values of a_{adj} for the simulation in figure 15. The initial state is chosen to be in the unsafe region. By applying positive a_{adj} , it is possible to move into the safe region in order to start the merging maneuver. The larger the value of a_{adj} is, the faster we move into the safe region. The minimum value of t_{adj} for each a_{adj} is determined by the point of intersection between the isocline that corresponds to the a_{adj} and the safety margin.

The longitudinal velocity of merging vehicle, v_M , cannot exceed an upper bound, which is determined by the capabilities of the vehicle and passenger comfort. Similar to the case of vehicles L_d and M , we may use the modified profile in figure 13, in the case where the acceleration profile of figure 10 requires velocities that exceed the aforementioned upper bound. Here again, we need to make sure that we are in the left half plane, $x_2 < 0$, and then we may set $a_{adj}=0$. Figure 17 shows the trajectory of moving from the unsafe region into the safe region.

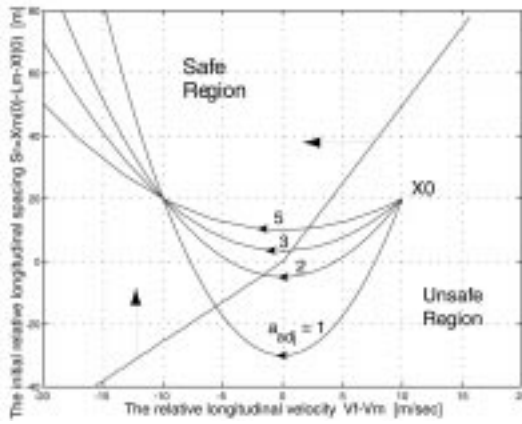


Fig 16: Applying a_{adj} to move from unsafe area into safe area.

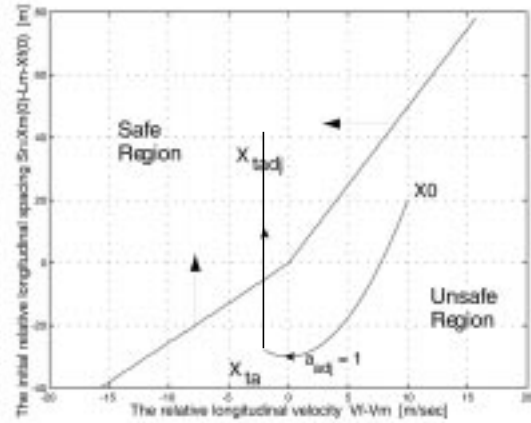


Fig 17: Applying modified a_{adj} to move from unsafe area into safe area.

Minimum Longitudinal Safety Spacing between M and L_o :

1) $t_{adj}=0$:

The velocity of all vehicles except M is assumed to be constant while the velocity of M becomes equal to the velocity of the destination lane after the time-instant t_{long} and remains constant thereafter, i.e. we assume $v_M(t)=v_{Ld}=v_{Fd}$ for t in the time interval $[t_{long}, T]$. The value of a_M is:

$$a_M = \begin{cases} \frac{v_{Lo} - v_M(0)}{t_{long}} & t \leq t_{long} \\ 0 & \text{Otherwise} \end{cases} \quad (53)$$

Without loss of generality we assume that $t_{long} > t_C$. According to equation (22) the condition for collision avoidance between M and L_o is:

$$Sr(t) = \left(Sr(0) + (v_{Lo} - v_M(0)) \times \left(t - \frac{t^2}{2t_{long}} \right) + (v_{Lo} - v_{Ld}) \times \frac{t^2}{2t_{long}} \right) > 0 \quad (54)$$

$$\forall t \in [0, t_C]$$

It should be noted that the last term in equation (54) is independent of v_M , but depends on the difference between the velocity of the vehicles in the originating and destination lanes. In order to find $MSS(L_o, M)$ analytically, we define the following variables:

$$\alpha = \frac{v_{Lo} - v_M(0)}{v_{Ld} - v_M(0)} \quad (55)$$

$$t_{max} = \alpha \times t_{long}$$

The above transformation is applicable provided that $v_{Ld} - v_M(0)$ is not zero. If $v_{Ld} - v_M(0)$ is zero, the value of α is zero, too. In this case, we can apply the results for the case of constant velocity, (subsection 3.1). Let us also define the following function,

$$S(t) = \text{Max} \left\{ \left((v_M(0) - v_{Lo}) \times \left(t - \frac{t^2}{2t_{long}} \right) + (v_{Ld} - v_{Lo}) \times \frac{t^2}{2t_{long}} \right), 0 \right\} \quad (56)$$

Using the above definitions we can analytically solve equation (69) as shown in Table 1.

Table 1: Analytical values of $MSS(L_o, M)$

IF	& IF	& IF	THEN
$\alpha < 0$	—	$v_{Ld} < v_M(0), v_{Lo} > v_M(0)$	$MSS(L_o, M) = S(0)$
"	—	$v_{Ld} > v_M(0), v_{Lo} < v_M(0)$	$MSS(L_o, M) = S(t_C)$
$\alpha > 0$	$t_{max} > t_C$	$v_{Ld} < v_M(0), v_{Lo} < v_M(0)$	$MSS(L_o, M) = S(t_C)$
"	"	$v_{Ld} > v_M(0), v_{Lo} > v_M(0)$	$MSS(L_o, M) = S(0)$
$\alpha > 0$	$t_{max} < t_C, t_{max} > t_C/2$	$v_{Ld} < v_M(0), v_{Lo} < v_M(0)$	$MSS(L_o, M) = S(t_{max})$
"	"	$v_{Ld} > v_M(0), v_{Lo} > v_M(0)$	$MSS(L_o, M) = S(0)$
$\alpha > 0$	$t_{max} < t_C/2$	$v_{Ld} < v_M(0), v_{Lo} < v_M(0)$	$MSS(L_o, M) = S(t_{max})$
"	"	$v_{Ld} > v_M(0), v_{Lo} > v_M(0)$	$MSS(L_o, M) = S(t_C)$

We performed simulations in order to demonstrate our theoretical results. In the simulations, we set $T=50$ sec, $t_{adj}=0$ sec, $t_{lat}=5$ sec, $H=12$ feet, and $t_{long}=10$ sec. The relative speed between the vehicles L_d and L_o was set equal to 20 and -20 miles/hour. The safe and unsafe regions are shown in figure 18. It should be noted that the less the relative velocity between the vehicles L_d and L_o is, the larger the safe region becomes. Comparison between figures 8 and 18 indicates that the slope of the safety margins remain almost the same, while there is a "horizontal shift" in the safety margins.

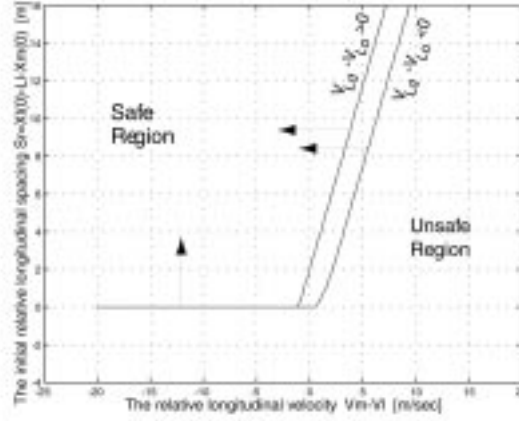


Fig 18: The collision region between M and Ld; Case II: switching longitudinal acceleration.

II) $t_{adj}>0$:

Similar to the previous cases we define the variables,

$$\begin{aligned} x_1 &= x_{Lo} - x_M - l_{L1} \\ x_2 &= v_M - v_{Lo} \end{aligned} \quad (57)$$

After obtaining the state space equations, the isoclines can be found as follows:

$$x_1 = -\frac{x_2^2}{2a_{adj}} + c \quad (58)$$

where constant c is a constant that depends on the initial values, $x_1(0)$ and $x_2(0)$. Figure 19 shows the isoclines corresponding to various values of a_{adj} . The initial point has been chosen to be in the unsafe region. By applying a_{adj} , it is possible to move into the safe region in order to start the merging maneuver. It should be noted that only two values of a_{adj} , i.e. -5, -7, are acceptable here. The other values result in $Sr(t)<0$, for some t in the interval $[0, t_{adj}]$, which cause collision between M and L_d (the shaded area in figure 19 corresponds to the negative values of $Sr(t)$ which are *infeasible*). The minimum value of t_{adj} for each a_{adj} is chosen to be the time-instant the

corresponding isocline curve intersects with the safety region.

Unfortunately in this case we can not easily apply the acceleration profile of figure 13. This is largely due to the existence of the infeasible region, and the small area bounded between the MSSs curve and the infeasible area. It should be noted that in the previous cases we use this area, where the relative longitudinal velocity between the merging vehicle M and the vehicles in the destination lane is negative, to move into before adjusting the longitudinal acceleration zero. In the case of positive relative velocity between L_d and L_o , i.e. $v_{Ld}-v_{Lo}>0$, it is unrealistic to exploit the modified switching acceleration profile, see figure 19.

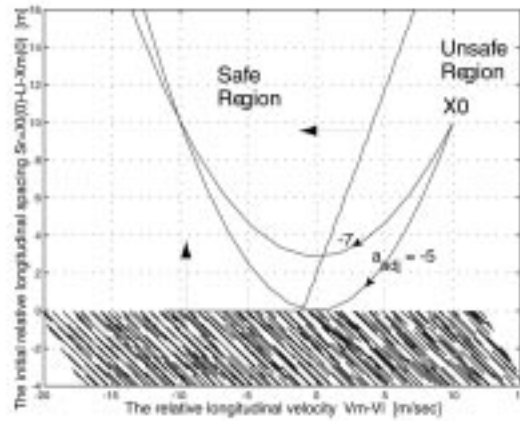


Fig 19: Applying a_{adj} to move from the unsafe region into the safe region.

Minimum Longitudinal Safety Spacing between M and F_o :

1) $t_{adj}=0$:

The velocity of all vehicles except M is assumed to be constant while the longitudinal velocity of M becomes equal to the longitudinal velocity of the vehicles in the destination line at the time-instant t_{long} and remains constant thereafter. Therefore the value of a_M is as same as that in equation (53). According to equation (29) the condition for collision avoidance between M and F_o is:

$$Sr(t) = \left(Sr(0) + (v_M(0) - v_{Fo}) \times \left(t - \frac{t^2}{2t_{long}} \right) + (v_{Ld} - v_{Fo}) \times \frac{t^2}{2t_{long}} \right) > 0 \quad (59)$$

$$\forall t \in [0, t_C]$$

It should be noted that the last term in equation (59) is dependent on the difference between the velocity of originating and destination lane. In order to find the minimum initial longitudinal safety spacing, $MSS(M, F_o)$, we define the following variables:

$$\alpha = \frac{v_M(0) - v_{Fo}}{v_M(0) - v_{Ld}} \quad (60)$$

$$t_{max} = \alpha \times t_{long}$$

It the relative velocity between two vehicles M and L_d , $v_M(0)-v_{Ld}$, happens to be zero in equation (60) we can apply the results of subsection 3.1. Let us define the variable $S(t)$ as follows:

$$S(t) = \text{Max} \left\{ \left((v_{Fo} - v_M(0)) \times \left(t - \frac{t^2}{2t_{long}} \right) + (v_{Fo} - v_{Ld}) \times \frac{t^2}{2t_{long}} \right), 0 \right\} \quad (61)$$

Using the above definitions we can analytically find $MSS(M, F_0)$ as shown in Table 2.

Table 2: Analytical values of $MSS(M, F_0)$

IF	& IF	& IF	THEN
$\alpha < 0$	—	$v_{Ld} < v_M(0), v_{Fo} > v_M(0)$	$MSS(M, F_0) = S(t_C)$
"	—	$v_{Ld} > v_M(0), v_{Fo} < v_M(0)$	$MSS(M, F_0) = S(0)$
$\alpha > 0$	$t_{max} > t_C$	$v_{Ld} < v_M(0), v_{Fo} < v_M(0)$	$MSS(M, F_0) = S(0)$
"	"	$v_{Ld} > v_M(0), v_{Fo} > v_M(0)$	$MSS(M, F_0) = S(t_C)$
$\alpha > 0$	$t_{max} < t_C, t_{max} > t_C/2$	$v_{Ld} < v_M(0), v_{Fo} < v_M(0)$	$MSS(M, F_0) = S(0)$
"	"	$v_{Ld} > v_M(0), v_{Fo} > v_M(0)$	$MSS(M, F_0) = S(t_{max})$
$\alpha > 0$	$t_{max} < t_C/2$	$v_{Ld} < v_M(0), v_{Fo} < v_M(0)$	$MSS(M, F_0) = S(t_C)$
"	"	$v_{Ld} > v_M(0), v_{Fo} > v_M(0)$	$MSS(M, F_0) = S(t_{max})$

For simulation we set $T=50$ sec, $t_{adj}=0$ sec, $t_{lat}=5$ sec, $H=12$ feet, and $t_{long}=10$ sec. The relative velocity between the vehicles L_d and F_o was set equal to 20 and -20 miles/hour. The safe and unsafe regions are shown in figure 20. Comparison between figures 9 and 20 indicates that the tangent of the safety margins remain almost the same, while again we observe a "horizontal shift" on the safety margins.

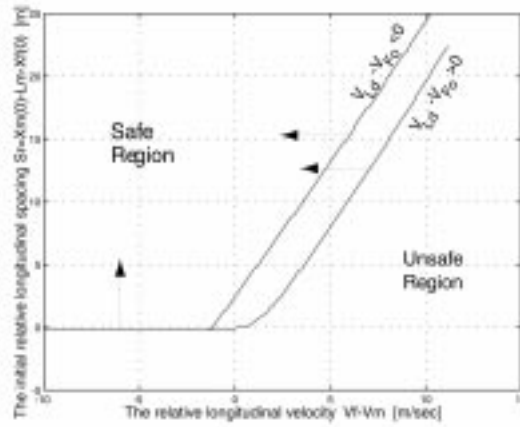


Fig 20: The collision region between Fo and M; Case II: switching acceleration.

II) $t_{adj} > 0$:

Here, the state space variable can be defined as follows:

$$\begin{aligned} x_1 &= x_M - x_{Fo} - l_M \\ x_2 &= v_{Fo} - v_M \end{aligned} \quad (62)$$

Obtaining the state space equations, the isoclines corresponding to a_{adj} can be found as follows:

$$x_1 = \frac{x_2^2}{2a_{adj}} + c \quad (63)$$

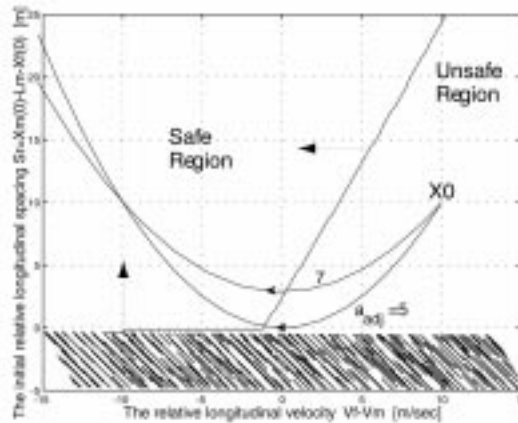


Fig 21: Applying a_{adj} to move from the unsafe region into the safe area.

where the constant c depends on the initial values, $x_1(0)$ and $x_2(0)$. Figure 21 shows the isoclines

corresponding to various values of a_{adj} . The initial point has been chosen to be in the unsafe region. By applying a_{adj} , it is possible to move into the safe region in order to start merging maneuver. It should be noted that only two values of a_{adj} , i.e. 5 and 7, are acceptable here. The other values result in $Sr(t) < 0$ that is collision between M and F_o . The minimum acceptable value of t_{adj} for each a_{adj} is determined by the point of intersection between the corresponding isocline curve and the safety margin.

4. Conclusions

In this paper, we analyzed the kinematics of the vehicles involved in a lane changing/merging maneuver, and studied the conditions under which lane changing/merging crashes can be avoided. That is, given a particular lane change/merge scenario, we calculated the minimum longitudinal spacing which the vehicles involved should initially have so that no collision, of any type, takes place during the maneuver.

Simulations of a number of examples were presented to demonstrate the results. We assumed in our simulations that all vehicles, involved in the lane changing/merging maneuver, were initially at steady state, i.e. their velocities were constant prior to the maneuver. Except for the merging vehicle, all vehicles keep their steady state velocity during the merging maneuver. Three different longitudinal acceleration scenarios - constant longitudinal velocity, switching longitudinal acceleration, and modified switching longitudinal acceleration - were applied to the merging vehicle in order to determine the safe and unsafe region as well as the MSS between the merging vehicle and its surrounding vehicles. We observed that the switching scenario and the modified switching scenario expanded the safe region for lane changing. Furthermore, by considering the longitudinal adjustment acceleration for the merging vehicle, we studied the possibility of moving from the unsafe region into the safe region. Our results together with appropriate sensors and equipment on the board of vehicles could be used to assess the safety of lane changing maneuvers and provide warnings or take evasive actions to avoid collision.

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