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Capacity at Unsignalized Intersections

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by R. Tapio Luttinen

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Abstract

This report presents a unified theoretical analysis of capacity at unsignalized intersections. Potential capacity equations for different headway and gap acceptance functions are presented following the results of earlier researchers. The capacity analysis for multiple independent major streams is based on the theory of superimposed point processes. A very general capacity equation is presented, which can also be applied in the Rank 3 capacity analysis.

Gap acceptance and queue discharge of a Rank 3 stream is described as a queuing model. It demonstrates that in the current methodology the probability of a queue-free state in Rank 2 streams should be modified. In addition, the headway distribution during a queue-free state should have a minimum headway equal to the follow-up time, and the flow rate should be adjusted accordingly. If the arrivals to an intersection are Poisson processes, the headways of Rank 2 streams during the queue-free periods follow shifted exponential distribution. The modified movement capacity equations are presented. The resulting capacities are higher than the HCM2000 capacities, especially at low traffic volumes.

In the analysis of Rank 4 capacity the dependency of Rank 3 queues on Rank 2 queues has been considered problematic. However, it is demonstrated that the v/c ratio of a Rank 3 stream j gives the probability that a stream j vehicle is being served (waiting for a gap or a follow-up time) given that there are no Rank 2 vehicles in the service system. Therefore, the v/c ratio contains the necessary information of this dependency. Thus, when the capacities of major streams are known, their relative priorities can be ignored in the capacity estimation of a minor flow. This makes the analysis much simpler, and the capacity equation for Rank 4 streams does not become too complicated. In fact, the same equation can be used to estimate the movement capacity of both Rank 3 and Rank 4 streams. The resulting capacities are higher than suggested by the HCM2000 method.

Preface

Most of the intersections in Finland (and elsewhere) are unsignalized. The vast majority of Finnish unsignalized intersections have yield signs on minor road approaches. Other unsignalized intersections are either stop controlled or uncontrolled, observing the “right before left” principle. The operational analysis of unsignalized intersections is of great significance both to highway authorities and road users.

This research has been funded by the Finnish Road Administration (Finnra). These results as well as field and simulation studies will be published as a separate report in the Finnra strategic project S12 (Solutions to improve main roads). The work has been coordinated by deputy director Pauli Velhonoja at Finnra Traffic Engineering. The discussions with research professor Urho Pulkkinen at VTT Industrial Systems are thankfully acknowledged.

The results in this report are theoretical. They will be used to develop a capacity analysis methodology for Finnish unsignalized intersections. It was, however, considered appropriate to make a separate presentation of the theoretical results for an evaluation by the community of traffic scientists.

The presentation is self-contained. The theory necessary to obtain the results is presented in the text. It is assumed that the reader is familiar with the basics of probability theory and the theory of stochastic processes. The level of mathematics is, however, kept as simple as possible. Intermediate results are used extensively to help a reader to better follow the line of thought. It would have been possible to make the presentation shorter, but it was considered appropriate to make a more full description of the models. In some cases alternative, and shorter, ways to obtain the results are presented in the text.

The text is focused on issues, where new results or ideas are presented. Many important issues have been omitted, because they are well documented in other sources.

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*Lahti, Finland
January 5, 2003*

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Glossary

The terminology of the *Highway Capacity Manual* (Transportation Research Board 2000), and American terminology in general, is followed. Most of the definitions below are taken from the Highway Capacity Manual. A major exception is the definition of a gap, which is similar to the definition presented by Drew (1968). See discussion on page 21.

Approach: A set of lanes at an intersection that accommodates all left-turn, through and right-turn movements from a given direction.

Base conditions: Characteristics for a given type of facility that are assumed to be the best possible from the point of view of capacity; that is, characteristics if further improved would not result in increased capacity. (Same as ideal conditions.)

Capacity: The maximum hourly rate at which persons or vehicles can reasonably be expected to traverse a point or uniform section of a lane or a roadway during a given time period under prevailing roadway, traffic, and control conditions.

Conflicting movements: The traffic streams in conflict at an intersection.

Critical gap: The minimum major-stream headway during which a minor-street vehicle can make a maneuver.

Delay: The additional travel time experienced by a driver, passenger or pedestrian.

Equilibrium conditions: The expected state of the system may have cyclic fluctuations, but otherwise is time independent. (Steady state conditions)

Exclusive turn lane: A designated left- or right-turn lane used only by vehicles making those turns.

Flow rate: The equivalent hourly rate at which vehicles pass over a given point or section of a lane or roadway during a given time interval less than one hour. Also “rate of flow”.

Follow-up time: Time between the departure of one vehicle from the minor street and the departure of the next vehicle using the same gap under a condition of continuous queuing.

Gap: A time interval between two consecutive major stream vehicles evaluated by a minor stream vehicle driver for a crossing or merging operation.

Gap acceptance: The process by which a minor stream vehicle accepts an available gap to maneuver.

Geometric delay: The component of delay that results when geometric features cause vehicles to reduce their speed in negotiating a facility.

Headway: Time between two vehicles passing a point measured from front bumper to front bumper.

Heavy vehicle: Any vehicle with more than four wheels touching the pavements during normal operation.

Ideal conditions: Characteristics for a given type of facility that are assumed to be the best possible from the point of view of capacity, that is, characteristics if further improved would not result in increased capacity. (Same as base conditions.)

Impedance: The reduction in the capacity of a minor stream, caused by the congestion of major streams at an unsignalized intersection.

Lag: Time interval between the arrival of a yielding vehicle and the passage of the next priority stream vehicle. (Forward waiting time)

Major road: The roadway of a higher importance in an intersection.

Major stream: A traffic stream to which the analyzed stream must give right-of-way. (Same as priority stream.)

Minor road: The road of lower importance at an unsignalized intersection, controlled by yield or stop signs.

Minor stream: A traffic stream which must give right-of-way to a higher priority stream.

Movement capacity: The capacity of a specific movement (stream) at an unsignalized intersection approach, assuming that the traffic has exclusive use of a separate lane.

Potential capacity: The capacity of a specific movement at a unsignalized intersection approach, assuming that it is unimpeded by other movements and has exclusive use of a separate lane.

Queue: A line of units (vehicles, bicycles, persons) waiting to be served.

Rank: A level of priority in the hierarchy of traffic streams at an unsignalized intersection.

Stochastic model: A mathematical model that employs random variables for at least one input parameter.

Stop time: A portion of control delay when vehicles are at complete stop. Also “stop delay”.

Through vehicles: All vehicles passing directly through an intersection and not turning.

Traffic condition: A characteristic of traffic flow, including distribution of vehicle types in the traffic stream, directional distribution of traffic, lane use distribution of traffic, and type of driver population on a given facility during the analysis period.

Volume to capacity ratio: The ratio of flow rate to capacity for a traffic facility. (v/c ratio)

Volume: The number of persons or vehicles passing a point on a lane, roadway, or other trafficway during some time interval.

Symbols and abbreviations

$B(\tau)$	Backward waiting time at time τ
b	Current duration of headway
C	Capacity
C_i	Capacity of stream i
$C_{a,i}$	Potential capacity of stream i during unimpeded periods
$C_{L,i}$	Potential capacity of minor stream i under limited priority
$C_{m,i}$	Movement capacity of minor stream i
$C_{p,i}$	Potential capacity of minor stream i
$\mathbb{E}[X]$	Expected value of random variable X
$f(x)$	Probability density function
$F(x)$	Cumulative distribution function; $\mathbb{P}\{X \leq x\}$
f_i	Impedance factor of a Rank 2 stream i
$f_{a,i}$	Fraction of time available for gap acceptance in stream i
f_L	Adjustment factor for limited priority merge
$h_{i,k}$	Departure headway of vehicle k in stream i
$\bar{h}_j$	Average departure headway of stream j
HCM	<i>Highway Capacity Manual</i>
I_k	The set of independent priority streams of minor stream k
i.i.d.	Independent and identically distributed
$N(\tau)$	Number of arrivals during time interval $(0, \tau]$
$\mathbb{P}\{A\}$	Probability of event A
p	Probability of arrival during a minimum time interval
$p_{0,i}$	Probability of empty system in stream i
p_i	Proportion of stream i in a set of streams
$p_{z,i}$	Probability of empty system in streams 1, 4, and i
q	Flow rate, traffic volume (veh/h)
$q_{a,i}$	Average stream i flow rate (veh/h) during free-flow periods
q_c	Circulating flow at a roundabout
q_i	Flow rate of movement i
q_{II}	Flow rate of a Rank 2 movement
q_{III}	Flow rate of a Rank 3 movement
q_M	Major (priority) stream flow rate
$q_{M,i}$	Major stream flow rate for minor stream i

$q(\tau)$	Flow rate at time τ
$R(x)$	Survival function; $R(x) = \mathbb{P}\{X > x\} = 1 - F(x)$
T	Headway (stochastic variable)
$T(\tau)$	Current headway at time τ
$T_{bc,i}$	Busy cycle in stream i
$T_{bp,i}$	Busy period in stream i
$T_{ip,i}$	Idle period in stream i
t	Length of a headway
t_i	Length of the i^{th} headway
t_c	Critical gap
t_f	Follow-up time
t_o	Shortest acceptable major stream headway in a flow-analogy model
t_p	Minimum headway between major-stream vehicles
v/c	Volume to capacity ratio
x_i	Average service time of stream i
$\alpha(t)$	Gap acceptance function
γ	Scale parameter of Cowan's M3 distribution
δ_i	Time of entry of minor stream vehicle i
θ	Scale parameter of shifted exponential distribution
λ_i	Flow rate (veh/s) of stream i
$\lambda_{a,i}$	Average stream i flow rate (veh/s) during free-flow periods
ϕ_i	Proportion of free headways in stream i
ρ_i	Utilization factor, v/c ratio of stream i
ρ'_i	v/c ratio of stream i
τ	Time
τ_i	Arrival time of major stream vehicle i
v	Length of lag
$Y(\tau)$	Lag (forward waiting time) at time τ
χ	Probability of antiblock and headway larger than follow-up time
$(a, b]$	Time interval $a < \tau \leq b$
$i \vee j$	Stream i or j
$i \wedge j$	Streams i and j
$\lfloor X \rfloor$	Floor function (greatest integer not larger than X)
X_+	Positive part of X ; i.e., $X_+ = (X + X)/2$

1

Introduction

At grade-separated intersections crossing movements are vertically separated. At signalized intersections (most) crossing movements are time separated. At unsignalized intersections each driver must find a safe moment for the movement observing current traffic, traffic signs (stop or yield) and pertinent regulations (“right before left”).

Most of the capacity analysis procedures of unsignalized intersections are based on stochastic models; i.e., *gap-acceptance theory*. Capacity is a function of the availability of critical gaps. A critical gap is the minimum time interval between successive major-stream vehicles, in which a minor-street vehicle can make a maneuver. This methodology has been developed especially in Germany by Harders (1968) and Siegloch (1973), without forgetting the theoretical foundation laid by Anglo-Saxon scientists, such as Major and Buckley (1962) and Tanner (1962). Priority stream headways are usually assumed to follow exponential distribution. This approach has been used in many capacity manuals, such as in Germany (Brilon, Großmann and Blanke 1994, Forschungsgesellschaft für Straßen- und Verkehrswesen 2001) and in the U.S. (Transportation Research Board 2000). The Swedish method (Statens vägverk 1977, Hanson 1978, Bergh 1991) uses four different headway distributions, based on number of lanes, distance from traffic signals, and type of movement. Roundabouts are analyzed as series of intersections or short merging zones. Gap-acceptance theory can be adjusted for conditions, where an approaching major stream vehicle slows down to give way to a merging minor stream vehicle. This type of operation is called “limited priority merge” (Troutbeck and Kako 1999, Troutbeck 1999).

In Britain capacity analysis has been based on statistical models; i.e., *empirical regression equations* (Kimber and Coombe 1980). This method is not sensitive to the headway distribution models, it gives a clear method to estimate the effect of intersection geometry, and can model heavily congested conditions under which the process of simple gap-acceptance “gives way to a more interactive one in which major road vehicles adjust their headways to allow minor road vehicles to enter” (Kimber and Coombe 1980).

A third approach is based on *traffic signal analogy* suggested by Winsten (in Beckmann, McGuire and Winsten 1956), and recently developed by Akçelik (1994). Time periods blocked by higher priority vehicles are considered as red intervals and time periods available for minor stream vehicles are considered as green periods. The Additive Conflict Flows (ACF) procedure (Brilon and Wu 2001, Brilon and Wu 2002) is similar to the traffic signal analogy. The ACF method has been developed to simplify the analysis of the hierarchy of priorities as well as the pedestrian and cyclist movements at unsignalized intersections.

The theoretical gap acceptance capacity of a minor stream crossing a major stream has been solved under different major stream headway distributions (Major and Buckley 1962, Tanner 1962, Plank 1982). The capacity of a minor road left-turn stream (Rank 4) at a four-leg intersection has received only approximative solutions without a solid theoretical foundation (Brilon and Wu 2002). It also appears that current equations for a Rank 3 (e.g., minor road through streams) capacity have some shortcomings in the estimation of the queue-free periods of Rank 2 streams and flow rates of these streams during the queue-free periods. The purpose of this report is to describe these problems and present theoretical solutions for the capacity of Rank 2, 3, and 4 streams at unsignalized intersections. The results are compared against the HCM2000 (Transportation Research Board 2000) methodology.

The gap acceptance theory is described in some detail. Specific attention is given to headway distributions so that all results needed in the capacity analysis are provided along with some other useful information. The discussion is focused on negative exponential, shifted exponential and Cowan's M3 distributions, which are the distributions most frequently used in capacity analyses.

The capacity under multiple independent streams provides equations for Rank 2 stream capacity crossing several streams, which do not have mutual conflicts. This problem has been analyzed by Tanner (1967) and Troutbeck (1986). The theory of superimposed point processes (Cox and Isham 1980) gives a more general framework for the analysis. These results also provide tools for the Rank 3 capacity analysis.

A queuing model for Rank 3 capacity analysis is described in some detail. This discussion reveals that current methods underestimate the fraction of queue-free time in Rank 2 streams and overestimate the Rank 2 flow rate during this fraction of time. Accordingly, HCM2000 underestimates the capacity of Rank 3 streams. Therefore also Rank 4 capacity estimates in HCM2000 are too low. Capacity equations for Rank 3 and Rank 4 streams are derived assuming that arrivals in all major streams are random (exponential headways).

The results in this report are theoretical. They will be used to develop a capacity analysis methodology for Finnish unsignalized intersections. It was, however, considered appropriate to present the theoretical results for an evaluation by the community of traffic scientists. This discussion has been focused on new theoretical ideas. Many important issues, such as two-stage crossing (Brilon, Wu and Lemke 1996, Luttinen 1998) and the effect of heavy vehicles, pedestrians, and bicycles, have been omitted, because they are well documented in other sources.

2

Gap acceptance theory

2.1 Relative priority of traffic streams

Unsignalized intersections can be either uncontrolled, yield-sign controlled, stop-sign controlled, or roundabouts. *Uncontrolled intersections* follow the “right before left” rule. Each vehicle must give way to vehicles approaching from the right. Left turning vehicles must give way to opposing traffic.

At *yield-sign controlled intersections* the yield-controlled approaches are referred to as minor road approaches. The approaches that are not controlled by yield signs are referred to as major road approaches. All minor road vehicles yield the right-of-way to major road vehicles. Left-turning vehicles in all approaches yield right-of-way to opposing through and right turning vehicles.

At *stop-sign controlled intersections*¹ the minor road approaches are controlled by stop signs. The principles of operation are otherwise similar to yield controlled intersections, but minor road vehicles must stop at the stop line before proceeding.

Roundabouts have a circulating roadway around a central island. All vehicles entering a roundabout must yield to circulating vehicles. All vehicles entering or exiting a roundabout make a right turn.

The theory describing the operations at an unsignalized intersection is called *gap acceptance theory*. It includes four basic elements (cf. Transportation Research Board 2000):

1. Relative priority of traffic streams
2. Distribution (availability) of gaps in higher priority streams
3. Usefulness of gaps to yielding vehicles (gap acceptance)
4. Queue discharge

¹HCM2000 (Transportation Research Board 2000) calls this type of intersection two-way stop-controlled (TWSC).

Availability of gaps is described by a probability distribution of headways in higher priority streams. A gap acceptance function describes the usefulness of headways as well as queue discharge. The relative priority of traffic streams will be discussed next.

In a simple case of an uncontrolled intersection of two one-way roads (Fig. 2.1) the south-bound traffic (q_{11}) must yield to east-bound traffic (q_2). The intersection operates in the same way, if the northern approach has a yield sign. (In Finland a yield sign is not used in this case, because it is not necessary.) If there is a stop sign, vehicles must stop before passing the sign. An intersection of two one-way roads is the simplest case to analyze, and it is used in the derivation of potential capacity.

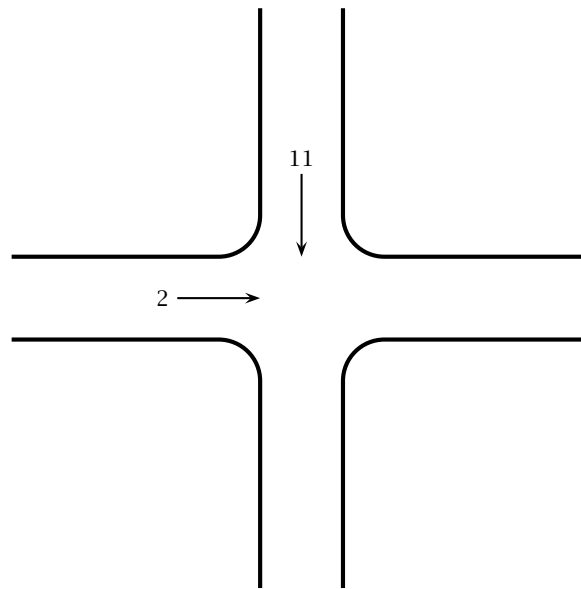


FIGURE 2.1 · Intersection of two one-way roads

The traffic streams at a four-leg intersection are presented in Figure 2.2. The numbering follows HCM2000 (Transportation Research Board 2000). Streams 13–16 are pedestrians crossing the road.

The operation is more complicated than in the simple case of Figure 2.1. All minor road vehicles yield the right-of-way to major road vehicles. Left-turning vehicles in all approaches yield right-of-way to opposing through and right-turning vehicles. There is a hierarchy of priorities. Through and right-turning movements on the major road (2, 3, 5, and 6) have absolute priority. This group of movements is called Rank 1. Left turning traffic from the major road (1, 4) and right turning traffic from the minor road (9, 12) have Rank 2. They must give priority to Rank 1 movements. Through traffic on the minor road (8, 11) has Rank 3. It must give priority to all movements on the major road (Ranks 1 and 2). Finally, left turning traffic from the minor road (7, 10) is subordinate to all other streams, and it has Rank 4.

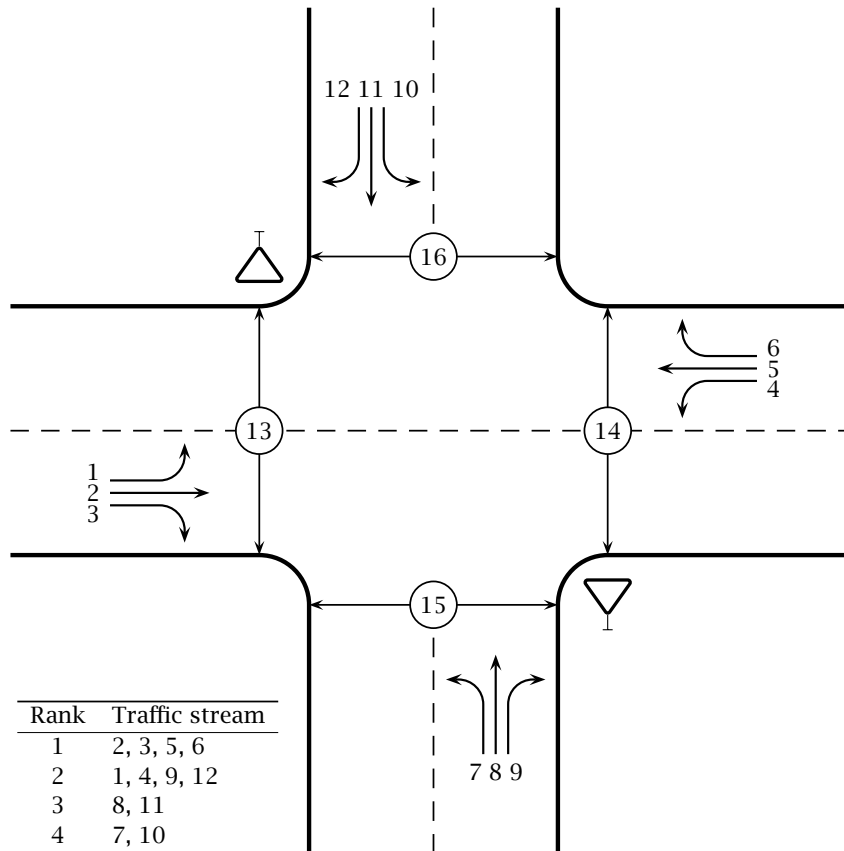


FIGURE 2.2 · Traffic streams at a four-leg intersection

If the major road has a wide central island, one or more minor street vehicles can be stored between the two directions of major road traffic streams. In this case minor road through or left-turn vehicles may operate in two stages, if there is storage space available (Brilon et al. 1996, Brilon and Wu 1999, Luttinen 1998)

In HCM2000 (Transportation Research Board 2000) pedestrian flows (15, 16) across the minor road have Rank 1 and pedestrian flows across the major road (13, 14) have Rank 2. HCM2000, however, states that the priority of pedestrian movements with respect to vehicular movements may be a policy issue varying by jurisdiction. In the Finnish guidelines (Tiensuunnittelutoimisto 1978) minor street movements are considered subordinate to pedestrian flows across the minor road. Pedestrian flows across the major road are ignored.—The capacity and level of service of pedestrian facilities is discussed e.g. in Chapter 18 of HCM2000 (Transportation Research Board 2000).

All yielding vehicles, which did not find an acceptable lag at arrival must decelerate and/or stop to wait for the next safe gap. There may, however, be also vehicles or pedestrians of higher rank waiting for an acceptable gap. Accordingly, a vehicle must find an acceptable gap or lag *after* all waiting higher rank vehicles and pedestrians have passed the conflict area.

Current Finnish highway code states that vehicles must give priority to a pedestrian who is on a crosswalk or is about to set foot on the crosswalk. A vehicle may not pass without stopping another vehicle that has stopped in front of a crosswalk. A turning vehicle must give priority to pedestrians crossing the road, where the driver is turning to. However, the priorities between pedestrians and vehicles observed in reality cannot be described with clear rules. Because pedestrians have a “higher” priority over turning vehicles than over through vehicles, pedestrian flow 15 has a higher priority over movement 3 (Rank 1) than over movement 8 (Rank 3). This breaks the hierarchical structure of priorities. In addition, these rules are often not observed by or even known to pedestrians and vehicle drivers (see also Brilon and Wu 2001, Brilon and Wu 2002). Consequently, the Rank of pedestrian flows is not defined in Figure 2.2. The effect of pedestrians should, however, be considered in the capacity calculations.

Right turning stream 3 do not have a conflict with minor road streams 7, 8 and, 9. It is, however, possible that minor road drivers are not confident that a major street vehicle will turn right, until they see a clear indication of its intentions. Accordingly, a stream may have an effect on the performance of another stream, even though no conflict exists between the two streams.

Major street through vehicles have absolute priority. They may, however, experience delays due to two reasons:

1. If major road through vehicles cannot pass queuing left-turn vehicles, they have to wait for the clearance of the queue (Kimber and Coombe 1980, Bonneson and Fitts 1997, Bonneson and Fitts 1999).
2. At high traffic volumes major road vehicles may be slightly delayed to accommodate minor street vehicles (Troutbeck 1990, Troutbeck and Kako 1997, Troutbeck and Kako 1999).

Wang (2000) has found that major and minor road traffic volumes do not have a significant effect on critical gaps. However, if major vehicles adjust their speeds to accommodate minor street vehicles, the result can be observed in the availability of gaps (headway distribution), not (primarily) in the gap acceptance behavior (see Troutbeck 1997, Troutbeck and Kako 1999).

The traffic streams of a T-intersection are presented in Figure 2.3. There are only three hierarchical levels of priorities (ranks). Consequently, there are less possible conflicts between streams, so that it is easier for a driver to observe the safe gaps in priority streams.

At an uncontrolled intersection all vehicles obey the “right-before-left” rule. It is possible to have a situation where priorities are unspecified. Such a situation occurs, when vehicles of movements 2, 4, and 7 simultaneously wait for an opportunity to proceed. Each vehicle must yield to another vehicle, none has priority over all others. The methods described in this report cannot be applied to uncontrolled intersections. The German manual (Forschungsgesellschaft für Straßen- und Verkehrswesen 2001) suggests 600–800 pc/h as the sum of maximum flow rates of all four approaches at an uncontrolled intersection.

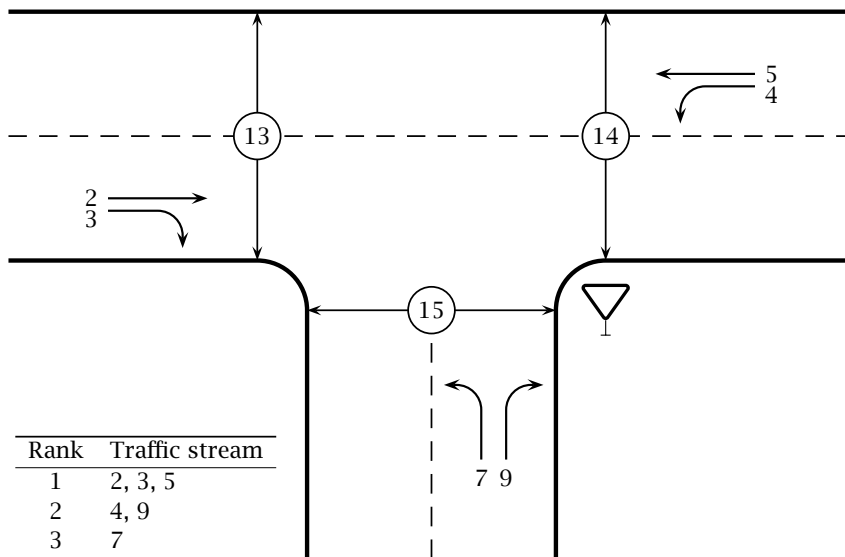


FIGURE 2.3 · Traffic streams at a T-intersection

At roundabouts (Fig. 2.4) vehicle circle counterclockwise. Approaching flows (q_a) give priority to circulating flows (q_c). This ensures an uninterrupted flow in the circulating roadway. Circulating and approaching flows merge immediately at the entrance to the circulating roadway. Each vehicle must make two right turns. All other movements are eliminated. As a subordinate vehicle enters the circulating roadway it becomes a priority vehicle.

Because vehicles entering the roundabout make a right turn, the operation can be assumed to be similar with the operation of right turning streams in yield (or stop) controlled intersections. The intersection angle is, however, usually smaller in roundabouts, which makes the operation similar to a (very) short merge area. If a roundabout is analyzed as a series of unsignalized intersections, the roundabout must be so large that the merges can be considered independent. According to the German manual (Forschungsgesellschaft für Straßen- und Verkehrswesen 2001), this approach can be used for roundabouts having an inscribed circle diameter larger than 26 meters.

2.2 Availability of gaps

2.2.1 Headways and lags

The time distance from front bumper to front bumper between two consecutive vehicles passing an observation point is called a *headway*. It is the sum of the time used by a vehicle to pass the observation point (occupancy time) and the time interval (gap) to the arrival of the next vehicle (Luttinen 1996). At unsignalized intersections the minor-road drivers consider gaps (unoccupied time intervals) as possible opportunities for their crossing or merging maneuvers. However,

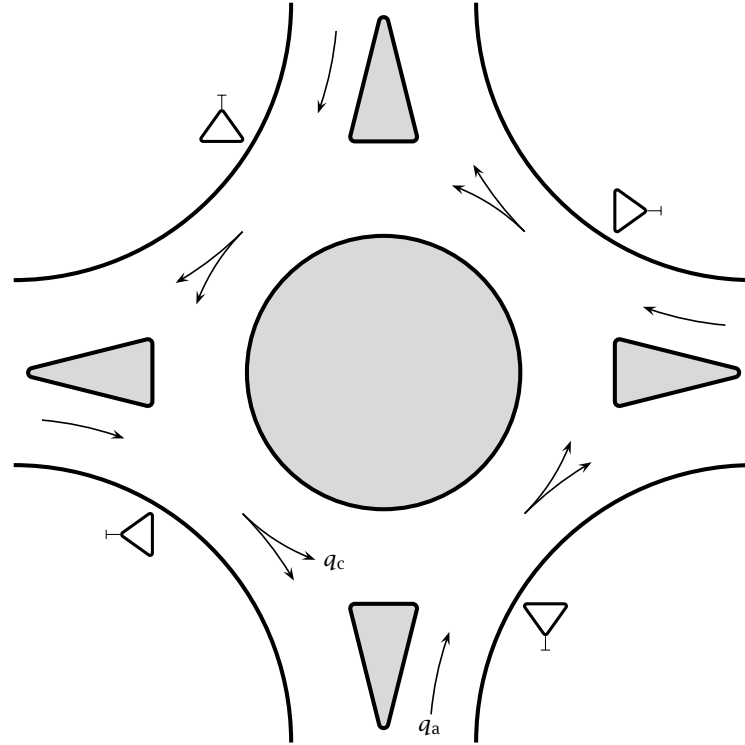


FIGURE 2.4 · Traffic streams at a roundabout

it is easier to describe traffic flow in terms of headways than in terms of gaps and occupancy times.² In this discussion *gap* is the time interval between two consecutive major stream vehicles evaluated by a minor stream driver for crossing or merging. The availability of gaps is described by headway distributions of higher priority streams. It is assumed that the differences in major stream vehicle lengths and speeds (i.e., occupancy times) can be ignored.

Mathematically the arrival of vehicles is described as a point process, where each arrival is a point in the time axis (Fig. 2.5). Because vehicle length is ignored, gaps and headways are equal. Headway (gap) t_n is the time interval between vehicles $n - 1$ and n . Each headway is connected to the vehicle which terminates the time interval. From the point of view of a minor-road driver it is thus possible to consider traffic flow on a major road as a succession of moving gaps (Raff 1950, Buckley 1962).

Let us assume that a vehicle enters a stop line or a yield line at time τ , the preceding priority vehicle has passed the conflict point at time τ_i , and the next priority vehicle will pass at time τ_{i+1} (Fig. 2.6). The time from the passage of

²Headway can be considered as the inverse individual flow rate of each vehicle (Dawson and Chimini 1968). The sum of headways $\tau_n = \sum_{i=1}^n t_i$ is the time interval between the arrivals of vehicles 0 and n , so that the number of headways (vehicles) divided by the sum of headways gives a proper estimate of flow rate ($\tilde{\lambda} = n\tau_n^{-1}$).—If vehicles are reduced to points, as mathematical models usually do, headways and gaps are equal.

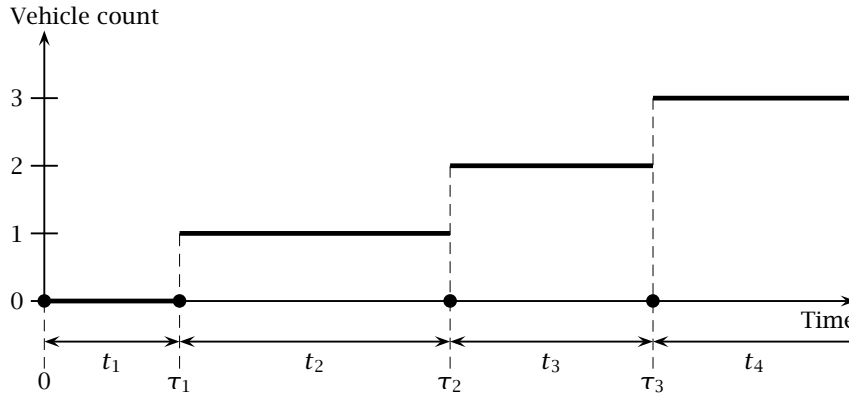
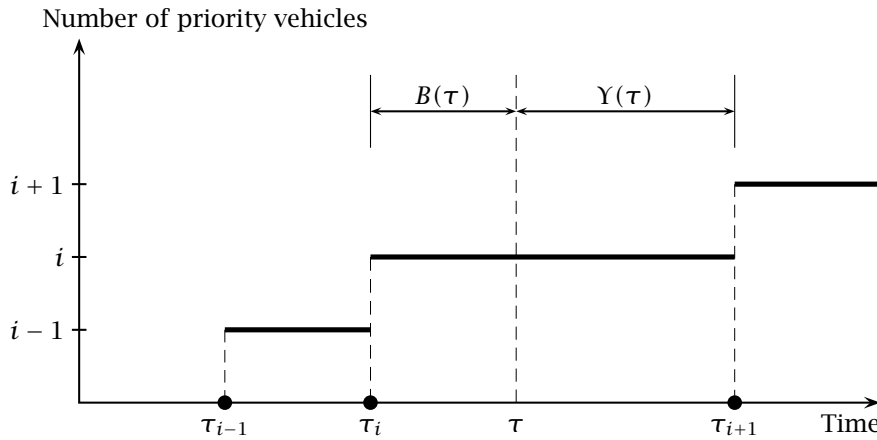


FIGURE 2.5 · Vehicle arrivals as a point process (Luttinen 1990)

the previous priority vehicle, $B(\tau) = \tau - \tau_i$, is called *backward waiting time* or *current life* (Karlin and Taylor 1975). The time to the passage of the next priority vehicle, $Y(\tau) = \tau_{i+1} - \tau$, is called *forward waiting time* or *excess life* or *lag*. The current headway at time τ is the headway of priority stream vehicle $i + 1$; i.e., $T(\tau) = T_{i+1} = \tau_{i+1} - \tau_i = B(\tau) + Y(\tau)$.

FIGURE 2.6 · Backward waiting time $B(\tau)$ and lag $Y(\tau)$ (Luttinen 1990)

Some of the major road gaps are not accepted by minor road vehicles, some gaps are accepted by one vehicle, and some gaps allow the entry of several vehicles. Consequently, more detailed information than a *deterministic headway model* ($\forall i: t_i = \bar{t}$) is required (see Watson 1933, Clayton 1941, Catchpole and Plank 1986). For realistic estimation of capacity and delays it is necessary to have a realistic model of the frequency of gaps of different lengths. In mathematical models the availability of gaps is described by a *probability distribution* of vehicle headways (see Luttinen 1996).

A *cumulative distribution function* gives the probability that a randomly³ selected headway is not larger than a given value

$$F(t) = \mathbb{P}\{T \leq t\} = \int_0^t f(u) \, d(u) \quad (2.1)$$

$$F(\infty) = \mathbb{P}\{T < \infty\} = \int_0^\infty f(u) \, d(u) = 1, \quad (2.2)$$

where $f(t)$ is (probability) *density function*. Because headways cannot have a negative length, the lower limit of integration is null. The probability that a randomly selected headway is larger than a given value is called *survivor function* or *survival function*

$$R(t) = \mathbb{P}\{T > t\} = \int_t^\infty f(u) \, d(u) = 1 - F(t). \quad (2.3)$$

In reliability and life data analysis this function gives the probability that an object survives or operates reliably for a given time (see e.g. Nelson 1982, Crowder, Kimber, Smith and Sweeting 1991).

The *expectation* $\mathbb{E}[T]$ is the average headway from the distribution:

$$\mathbb{E}[T] = \bar{t} = \int_0^\infty t f(t) \, dt. \quad (2.4)$$

Because $f(t) = 0$ when $t < 0$, expectation can also be expressed in terms of the survival function (Grimmett and Stirzaker 2001):⁴

$$\mathbb{E}[T] = \bar{t} = \int_0^\infty [1 - F(t)] \, dt = \int_0^\infty R(t) \, dt. \quad (2.5)$$

If we count n consecutive headways, the expected duration of the count is $\mathbb{E}[\tau_n] = n\bar{t}$. Flow rate ($\lambda = q/3600$) is the inverse of average headway: $\lambda = n(\mathbb{E}[\tau_n])^{-1} = \bar{t}^{-1}$. This is an unconventional definition of flow rate, but it helps to demonstrate the relationship between flow rate and average headway. For a more detailed analysis of this relationship the reader is referred to the discussions by Haight (1963) on synchronous and asynchronous counting and by Luttinen (2001)

³It should be emphasized that the headway that occurs for a randomly arriving vehicle or pedestrian is not randomly selected. It is more probable that a random observer arrives during a long headway than during a short headway. This length-biased sampling is in the core of the well-known waiting time paradox (Feller 1966, Daganzo 1997).

⁴This equation can be proved by integration with change of order (Ross 2002)

$$\begin{aligned} \int_0^\infty R(t) \, dt &= \int_0^\infty \int_t^\infty f(u) \, du \, dt \\ &= \int_0^\infty \left(\int_0^u dt \right) f(u) \, du \\ &= \int_0^\infty u f(u) \, du. \end{aligned}$$

For a geometric interpretation see Gnedenko (1963).

on doubly synchronous counting as well as to a discussion on renewal processes in any standard textbook on stochastic processes (e.g., Karlin and Taylor 1975).

The *lags* experienced by randomly arriving vehicles have a different distribution than headways. If $R(u) = \mathbb{P}\{T > u\}$ is the survival function of the headway distribution and $\mathbb{E}[T]$ is the expected headway, the probability density function of lags can be expressed using the *Palm-Khintchine equation* (Palm 1943, Khintchine 1960, Cox and Isham 1980):

$$f_Y(u) = \frac{R(u)}{\mathbb{E}[T]} = \lambda R(u). \quad (2.6)$$

Cox and Miller (1965) have presented an application of this equation to the estimation of delays at unsignalized intersections.

The survival function of lags is obtained directly from equation (2.6) as

$$R_Y(u) = \lambda \int_u^\infty R(y) dy. \quad (2.7)$$

The expected value of a lag can be expressed in terms of the moments of the headway distribution:

$$\begin{aligned} \mathbb{E}[Y] &= \int_0^\infty u f_Y(u) du \\ &= \frac{1}{\mathbb{E}[T]} \int_0^\infty u R(u) du \\ &= \frac{1}{\mathbb{E}[T]} \int_0^\infty u \int_u^\infty f(y) dy du \\ &= \frac{1}{\mathbb{E}[T]} \int_0^\infty \left(\int_0^y u du \right) f_T(y) dy \\ &= \frac{1}{\mathbb{E}[T]} \int_0^\infty \frac{y^2}{2} f(y) dy \\ &= \frac{\mathbb{E}[T^2]}{2 \mathbb{E}[T]} = \frac{\bar{t}}{2} + \frac{\sigma_T^2}{2\bar{t}}. \end{aligned} \quad (2.8)$$

See Kleinrock (1975) for a different approach.

2.2.2 Geometric distribution

Winsten (in Beckmann et al. 1956) has estimated the delay at an unsignalized intersection considering time in finite sized steps. The arrival process was binomial, and headways followed *geometric distribution*, which has density, distribution and survival functions as follows:

$$\begin{aligned} f(k) &= \mathbb{P}\{K = k\} = (1 - p)^k p \\ F(k) &= \mathbb{P}\{K \leq k\} = p \sum_{i=0}^k (1 - p)^i = 1 - (1 - p)^{k+1} \\ R(k) &= \mathbb{P}\{K > k\} = 1 - F(k) = (1 - p)^{k+1}, \end{aligned} \quad (2.9)$$

where p is the probability of arrival during the minimum time interval, and K is the headway (measured in discrete time intervals). This is a discrete-time model. Modern models are, however, based on continuous-time headway distributions.

2.2.3 Negative exponential distribution

If vehicles are assumed to move with their desired speeds and independently of each other, traffic volumes follow Poisson distribution, and headways are *exponentially distributed*. The probability density, distribution, and survival functions are

$$\begin{aligned} f(t) &= \lambda e^{-\lambda t} \\ F(t) &= 1 - e^{-\lambda t} \\ R(t) &= 1 - F(t) = e^{-\lambda t}, \end{aligned} \quad (2.10)$$

where λ is the scale parameter (see Fig. 2.7). The expected headway is equal to the inverse scale parameter:

$$\mathbb{E}[T] = \int_0^{\infty} R(t) dt = \int_0^{\infty} e^{-\lambda t} dt = \frac{1}{\lambda}. \quad (2.11)$$

This indicates that the scale parameter is equal to the flow rate in veh/s; i.e., $\lambda = q/3600$.

Negative exponential distribution is the interarrival time distribution of totally random arrivals. The counting process follows the Poisson distribution (Gerlough 1955). At low flow rates the negative exponential density function is highest, and distribution function (Fig. 2.7) rises steeply. For long headways the density approaches asymptotically zero and the distribution function unity. This indicates both a substantially high probability of very short headways, even at low flow rates, and the possibility of very long headways, even at high flow rates.

Several authors (e.g., Weiss and Herman 1962, Breiman 1963, Thedéen 1964) have demonstrated that under rather weak assumptions (no vehicle interaction, identically and independently distributed speeds) the number of vehicles in an arbitrary time interval will be asymptotically Poisson distributed as time tends to infinity. Thus, headways will asymptotically follow the exponential distribution. If the combined headways of several independent streams passing a reference point are considered, the limitations of the exponential distribution become less severe (see section 3.1.2).

Exponential distribution has the Markov (“lack of memory”) property (Luttinen 1990):

$$\begin{aligned} \mathbb{P}\{T > b + v \mid T > b\} &= \frac{e^{-\lambda(b+v)}}{e^{-\lambda b}} \\ &= e^{-\lambda v} \\ &= \mathbb{P}\{T > v\}, \end{aligned} \quad (2.12)$$

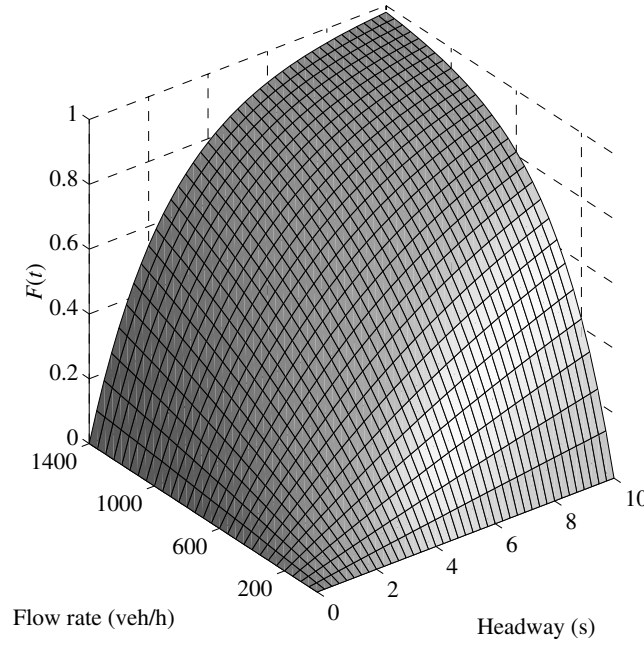


FIGURE 2.7 · Negative exponential cumulative distribution function for vehicle headways

where b is the current duration of headway (backward waiting time) and v is the lag (Fig. 2.6). This indicates that lags have the same distribution as headways:

$$f_Y(v) = \frac{R_T(v)}{\mathbb{E}[T]} = \lambda e^{-\lambda v}. \quad (2.13)$$

In fact, exponential distribution is the only continuous-time distribution that has the Markov property (Feller 1957).

Exponential distribution lends itself to demanding mathematical analyses. Therefore, it is often preferred over other, more realistic but also more complex distributions. In many cases explicit solutions can only be found when the arrival process is assumed Poisson. Since Adams (1936) the negative exponential distribution has played a central role in traffic flow theory.

The exponential model has two major limitations: The model allows unrealistically short headways, and it does not describe platooning. The model gets more distorted as flow rates increase. Consequently, the exponential distribution can be considered as a headway model for very low flow conditions, approximately $q < 150$ veh/h (Luttinen 1996). The exponential distribution can also be used to describe the headways of free vehicles, not driving in platoons. This indicates that empirical headway distributions have an “exponential tail” (Luttinen 1996).

2.2.4 Shifted exponential distribution

Shifted exponential distribution assumes that headways have a minimum length of t_p , and headways larger than t_p follow exponential distribution with probability density

$$f(t) = \begin{cases} 0, & \text{if } t < t_p \\ \theta e^{-\theta(t-t_p)}, & \text{if } t \geq t_p, \end{cases} \quad (2.14)$$

where θ is the scale parameter and t_p is the location parameter. Shifted exponential density can also be expressed using the unit (Heaviside) step function as

$$f(t) = \theta e^{-\theta(t-t_p)} u(t - t_p), \quad (2.15)$$

where (Spanier and Oldham 1987)

$$u(t - t_p) = \begin{cases} 0, & \text{if } t < t_p \\ \frac{1}{2}, & \text{if } t = t_p \\ 1, & \text{if } t > t_p. \end{cases} \quad (2.16)$$

The distribution function (Fig. 2.8) is

$$F(t) = \begin{cases} 0, & \text{if } t < t_p \\ 1 - e^{-\theta(t-t_p)}, & \text{if } t \geq t_p, \end{cases} \quad (2.17)$$

and the survival function

$$R(t) = \begin{cases} 1, & \text{if } t < t_p \\ e^{-\theta(t-t_p)}, & \text{if } t \geq t_p. \end{cases} \quad (2.18)$$

Shifted exponential distribution is the conditional distribution of exponential headways larger than t_p :

$$\mathbb{P}\{T > t \mid T > t_p\} = \frac{\mathbb{P}\{T > t\}}{\mathbb{P}\{T > t_p\}} = \frac{e^{-\lambda t}}{e^{-\lambda t_p}} = e^{-\lambda(t-t_p)}, \quad t_p \leq t. \quad (2.19)$$

This distribution avoids the problem of extremely short headways, but does not model platooning.

The shape of the distribution function is similar to the negative exponential distribution (Fig. 2.7), but the probability of headways smaller than the minimum headway (t_p) is null. After the minimum headway the distribution function rises even steeper than the negative exponential distribution function, and approaches asymptotically unity. The model does not include platooning, but the density is highest at headways just larger than the minimum headway.

The expected headway is

$$\mathbb{E}[T] = \int_0^\infty R(t) dt = \int_0^{t_p} 1 dt + \int_{t_p}^\infty e^{-\theta(t-t_p)} dt = t_p + \frac{1}{\theta}. \quad (2.20)$$

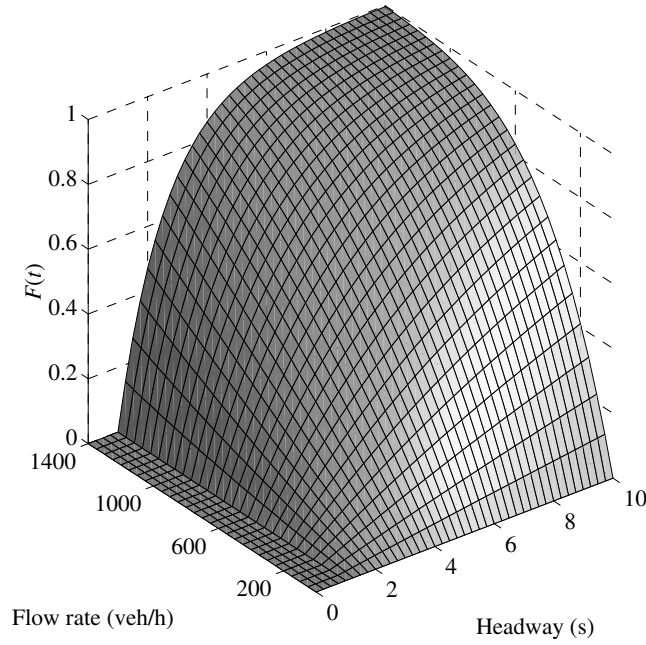


FIGURE 2.8 · Shifted exponential cumulative distribution function with minimum headway $t_p = 1$ s

Scale parameter θ can now be expressed in terms of flow rate as

$$\theta = \frac{1}{\mathbb{E}[T] - t_p} = \frac{\lambda}{1 - \lambda t_p} = \frac{q}{3600 - q t_p}. \quad (2.21)$$

The average flow rate of an arrival process with shifted exponential time headways can now be expressed as

$$\lambda = \frac{1}{\mathbb{E}[T]} = \frac{\theta}{1 + \theta t_p}. \quad (2.22)$$

Oliver (1961) has derived the explicit counting distribution.

The density function of the lag distribution is

$$f_Y(v) = \frac{R(v)}{\mathbb{E}[T]} = \frac{\theta R(v)}{1 + \theta t_p} = \begin{cases} \frac{\theta}{1 + \theta t_p}, & \text{if } v < t_p \\ \frac{\theta e^{-\theta(v-t_p)}}{1 + \theta t_p}, & \text{if } v \geq t_p. \end{cases} \quad (2.23)$$

The survival function of the lag distribution can be obtained by integrating the

density function. When $v < t_p$, the survival function is

$$\begin{aligned} R_Y(v) &= \frac{\theta}{1 + \theta t_p} \int_v^{t_p} dy + \frac{1}{1 + \theta t_p} \int_{t_p}^{\infty} \theta e^{-\theta(v-t_p)} dy \\ &= \frac{\theta(t_p - v)}{1 + \theta t_p} + \frac{1}{1 + \theta t_p} \\ &= 1 - \frac{\theta v}{1 + \theta t_p}, \quad v < t_p. \end{aligned} \quad (2.24)$$

When $v \geq t_p$, the survival function is

$$\begin{aligned} R_Y(v) &= \frac{1}{1 + \theta t_p} \int_v^{\infty} \theta e^{-\theta(v-t_p)} dy \\ &= \frac{e^{-\theta(v-t_p)}}{1 + \theta t_p}, \quad v \geq t_p. \end{aligned} \quad (2.25)$$

The survival function can now be expressed as

$$R_Y(v) = \begin{cases} 1 - \frac{\theta v}{1 + \theta t_p}, & \text{if } v < t_p \\ \frac{e^{-\theta(v-t_p)}}{1 + \theta t_p}, & \text{if } v \geq t_p. \end{cases} \quad (2.26)$$

When the location parameter (minimum headway) is zero, we get the survival function of negative exponential distribution, as expected.

Once the survival function is known, the expected value of a lag is obtained as

$$\begin{aligned} \mathbb{E}[Y] &= \int_0^{\infty} R_Y(v) dv \\ &= \int_0^{t_p} \left(1 - \frac{\theta v}{1 + \theta t_p}\right) dv + \frac{1}{1 + \theta t_p} \int_{t_p}^{\infty} e^{-\theta(v-t_p)} du \\ &= t_p - \frac{\theta t_p^2}{2(1 + \theta t_p)} + \frac{1}{\theta(1 + \theta t_p)} \\ &= \frac{1}{2\theta} \left((1 + \theta t_p) + \frac{1}{(1 + \theta t_p)} \right). \end{aligned} \quad (2.27)$$

Equation (2.8) gives naturally the same result, when the substitutions for mean $\bar{t} = t_p + \theta^{-1}$ and variance $\sigma_{\bar{t}}^2 = \theta^{-2}$ of the shifted exponential headway distribution are made. When $t_p = 0$, the result is equal to the expectation of negative exponential distribution (θ^{-1}).

2.2.5 Erlang distribution

Another simple model used in the analysis of unsignalized intersections is the *Erlang distribution* (Prevedouros 1988), which is a special case of the *gamma distribution*. It is easily tractable in mathematical analysis, but not as easy as the exponential distribution, and not particularly good as a headway model (Luttinen 1991, Luttinen 1996). The Erlang distribution has not been used in this research.

2.2.6 Cowan's M3 distribution

A logical step is to describe headway distribution as a mixture of follower and free-vehicle headways. If headways in platoons are assumed to be constant (t_p) and free-vehicle headways follow shifted exponential distribution, the cumulative distribution function is obtained as (see Luttinen 1990)

$$F(t) = \begin{cases} 0, & \text{if } t < t_p \\ 1 - \phi e^{-\gamma(t-t_p)}, & \text{if } t \geq t_p, \end{cases} \quad (2.28)$$

where ϕ is the proportion of free-vehicle headways, and γ is the scale parameter. This distribution is called Cowan's M3 (Model 3) distribution. The survival function is

$$R(t) = \begin{cases} 1, & \text{if } t < t_p \\ \phi e^{-\gamma(t-t_p)}, & \text{if } t \geq t_p, \end{cases} \quad (2.29)$$

The expected headway is

$$\mathbb{E}[T] = \int_0^\infty R(t) dt = \int_0^{t_p} 1 dt + \phi \int_{t_p}^\infty e^{-\gamma(t-t_p)} dt = t_p + \frac{\phi}{\gamma}. \quad (2.30)$$

Maximum flow rate $q = 3600t_p^{-1}$ is attained when all vehicles move in platoons ($\phi = 0$).

Because the proportion $1 - \phi$ of vehicles are followers and have a headway of t_p , the density function has a mass of $1 - \phi$ at t_p . This point mass can be described using a unit impulse (Dirac delta) function:

$$\delta(t - t_p) = \begin{cases} \infty & \text{if } t = t_p \\ 0 & \text{otherwise} \end{cases} \quad \text{and} \quad \int_0^\infty \delta(t - t_p) dt = 1. \quad (2.31)$$

The density function is now (Plank 1982, Luttinen 1999)

$$f(t) = (1 - \phi)\delta(t - t_p) - \phi\gamma e^{-\gamma(t-t_p)}. \quad (2.32)$$

Figure 2.9 displays the coefficient of variation

$$C(T) = \frac{\sqrt{\text{var}(T)}}{\mathbb{E}[T]} = \frac{\sqrt{\phi(2 - \phi)}}{\gamma t_p + \phi} \quad (2.33)$$

of Cowan's M3 distribution. It is null for deterministic headways ($\phi = 0$), reaches the maximum $[\gamma t_p(2 + \gamma t_p)]^{-1/2}$ at $\phi = \gamma t_p(1 + \gamma t_p)^{-1}$, and decreases to $(1 + \gamma t_p)^{-1}$ as ϕ reaches unity and the distribution becomes shifted exponential (Luttinen 1999). At low values of γ the peak rises above unity, but it stays below unity when γ is large. Although M3 gives a very crude approximation of short headways, it can produce moment characteristics very similar to real headway distributions (Luttinen 1999).

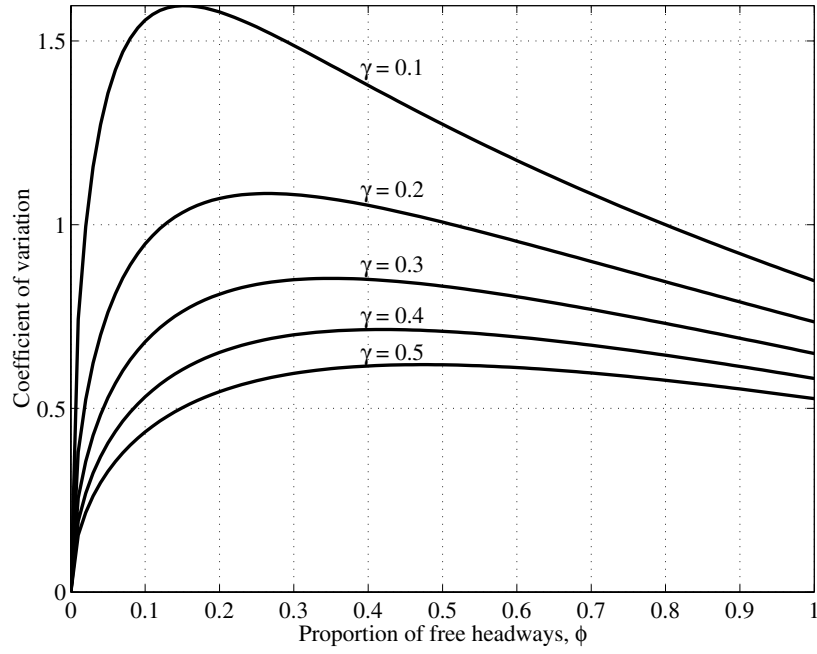


FIGURE 2.9 · Coefficient of variation of Cowan's M3 distribution with $t_p = 1.8$ s (Luttinen 1999)

The density function of the lag distribution is

$$f_Y(u) = \frac{R(u)}{\mathbb{E}[T]} = \frac{\gamma R(u)}{\phi + \gamma t_p} = \begin{cases} \frac{\gamma}{\phi + \gamma t_p}, & \text{if } u < t_p \\ \frac{\phi \gamma e^{-\gamma(u-t_p)}}{\phi + \gamma t_p}, & \text{if } u \geq t_p. \end{cases} \quad (2.34)$$

The survival function of the lag distribution can be obtained by integrating the density function. When $u < t_p$, the survival function is

$$\begin{aligned} R_Y(u) &= \frac{\gamma}{\phi + \gamma t_p} \int_u^{t_p} dy + \frac{1}{\phi + \gamma t_p} \int_{t_p}^{\infty} \gamma e^{-\gamma(u-t_p)} dy \\ &= \frac{\gamma(t_p - u) + 1}{\phi + \gamma t_p}, \quad u < t_p. \end{aligned} \quad (2.35)$$

When $u \geq t_p$, the survival function is

$$\begin{aligned} R_Y(u) &= \frac{\phi}{\phi + \gamma t_p} \int_u^{\infty} \gamma e^{-\gamma(u-t_p)} dy \\ &= \frac{\phi e^{-\gamma(u-t_p)}}{\phi + \gamma t_p}, \quad u \geq t_p. \end{aligned} \quad (2.36)$$

The survival function can now be expressed as

$$R_Y(u) = \begin{cases} \frac{\gamma(t_p - u) + 1}{\phi + \gamma t_p}, & \text{if } u < t_p \\ \frac{\phi e^{-\gamma(u - t_p)}}{\phi + \gamma t_p}, & \text{if } u \geq t_p. \end{cases} \quad (2.37)$$

When the proportion (ϕ) of free headways is unity, we obtain the survival function of shifted exponential distribution.

A model similar to Cowan's M3 was presented earlier by Tanner (1953, 1961*a*, 1967) and (Miller 1961). Buckley (1962) called it a regular-random distribution, but nowadays it is most often known as Cowan's M3 distribution according to (Cowan 1975). It describes the discharge flow from an M/D/1 queuing system, but unlike in the M/D/1 output the headways of the M3 distribution are assumed to be independent and identically distributed (i.i.d.), which means that platoon lengths follow geometric distribution. The M3 distribution has been widely used in the capacity and delay analysis of unsignalized intersections and roundabouts (Akçelik 1994, Hagring 1996, Hagring 1998, Sullivan and Troutbeck 1994, Troutbeck 1986, Troutbeck 1991, Troutbeck and Kako 1997, Luttinen 1999), especially in Australia.

Tanner (1953, 1961*a*) has modeled traffic flow as departures from an M/D/1 queuing system (exponential interarrival times and one server with constant service times) the proportion of constant (t_p) headways is equal to the utilization factor of the server (Luttinen 1990)

$$1 - \phi = \mathbb{P}\{T = t_p\} = \lambda t_p = \frac{q t_p}{3600}. \quad (2.38)$$

Using equations (2.30) and (2.38) parameters ϕ and γ can be expressed as

$$\phi = 1 - \lambda t_p = 1 - \frac{q t_p}{3600} \quad (2.39)$$

$$\gamma = \frac{\phi}{\frac{1}{\lambda} - t_p} = \lambda = \frac{q}{3600}. \quad (2.40)$$

The last result indicates that the scale parameter γ of the shifted exponential distribution of free headways is equal to the scale parameter λ of the negative exponential interarrival time distribution of the underlying M/D/1 queuing process. The headways are no longer independent; i.e., platoon lengths do not follow geometric distribution but the Borel-Tanner distribution (Tanner 1953, Tanner 1961*a*, Tanner 1961*b*, Haight and Breuer 1960, Prabhu 1965). The autocorrelation of headways does not have any influence on capacity estimates, but delays are sensitive to the order in which headways arrive (Kyte, Tian, Mir, Hameedman-soor, Kittelson, Vandehey, Robinson, Brilon, Bondzio, Wu and Troutbeck 1996).

Figure 2.10 displays the cumulative distribution function of headways from an M/D/1 queuing process with service time (i.e., headway in platoon) $t_p = 2$ s. There is a discontinuity at $T = t_p$. The probability of shorter headways is null. The proportion of follower headways increases linearly with increasing flow rate, as

indicated by equation (2.39). At headways larger than t_p the distribution function approaches asymptotically unity. Maximum flow rate is $\lambda = t_p^{-1}$ or $q = 3600t_p^{-1}$, when all vehicles are followers ($\phi = 0$).

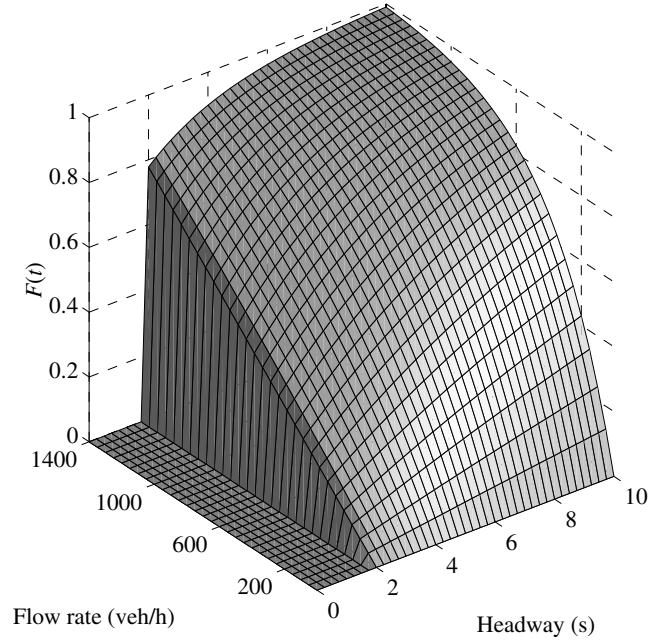


FIGURE 2.10 · Cumulative distribution function of headways from an M/D/1 queuing process with service time $t_p = 2$ s

M3 distribution is a good headway model, if there is no need for accurate modeling of short headways. Such is the case in the analysis of unsignalized intersections. M3 can reproduce data with moment characteristics very similar to real headway distributions, and it gives good results in capacity analysis for unsignalized intersections (Luttinen 1999). Properties of the M3 distribution and the estimation of its parameters have been discussed by several authors (see Brilon 1988a, Akçelik and Chung 1994, Sullivan and Troutbeck 1994, Hagring 1996, Troutbeck 1997, Luttinen 1999).

More advanced mixed distributions, such as the *semi-Poisson distribution* (Buckley 1962, Buckley 1968, Luttinen 1994) and the *generalized queuing model* (Cowan 1975, Branston 1976), use a specific distribution for headways in platoons and exponential distribution for free-vehicle headways. These models are more realistic, but they are also more difficult to use in mathematical analysis (Luttinen 1996).

Most of the current unsignalized intersection models use either negative exponential distribution or Cowan's M3 distribution for major stream headways. For a more detailed description of headway distributions the reader is referred to Luttinen (1996).

2.3 Gap acceptance and queue discharge

The arrivals and departures at an unsignalized intersection can be described by a queuing model demonstrated in Figure 2.11. When a vehicle enters the stop line or yield line (first position in a queue, time 0 in Figure 2.11) the driver must evaluate whether the lag in priority streams is safe for entry. If the driver considers the lag unsafe, the vehicle waits for a safe gap. The minimum acceptable major stream headway is called critical acceptance gap, or simply the *critical gap* (t_c).

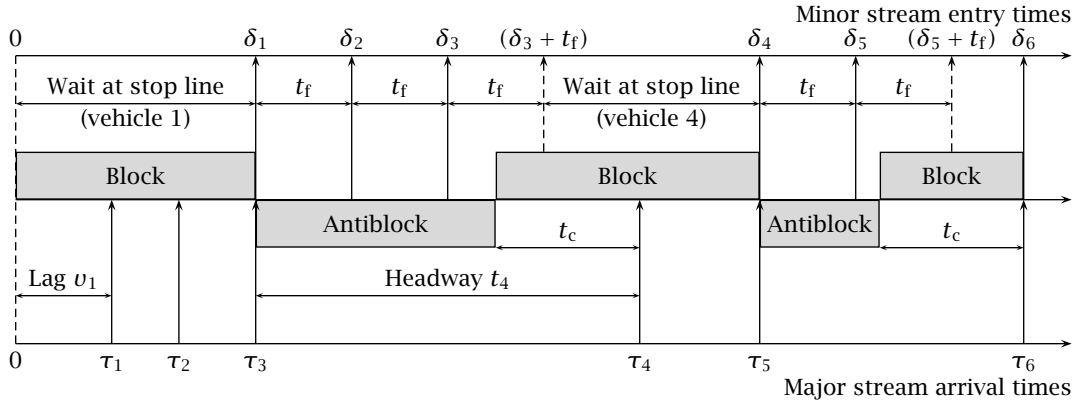


FIGURE 2.11 · Queue discharge at an unsignalized intersection

Each major stream vehicle blocks minor stream vehicles from entering the intersection. The *block*, as Raff (1950) called this time interval, starts t_c seconds before the major stream vehicle arrives and lasts until the beginning of the next major stream headway $t_i \geq t_c$ (Fig. 2.11). The time interval between two blocks is called an *antiblock*. An antiblock starts when a headway $t_i \geq t_c$ starts and ends t_c before the arrival of the next major stream vehicle. Minor stream vehicles can enter the intersection during antiblocks. Following a traffic signal analogy suggested first by Winsten (in Beckmann et al. 1956) blocks can be considered red intervals and antiblocks green intervals.

When a minor stream vehicle enters the intersection it takes some time before the next vehicle can take its position at the head of the queue. After this the process is repeated. The minimum headway between two minor stream vehicles entering the same major stream gap is called *follow-up time* (t_f). During an antiblock vehicles from a minor stream queue enter the intersection separated by follow-up times.⁵ An interval t_f after the queue has discharged minor stream vehicles can proceed without delay until the next block starts. At a yield controlled intersection a vehicle approaching an intersection or discharging from a queue can proceed without stopping, if the lag is long enough.

⁵According to Akçelik (1994) the first minor stream vehicle enters an intersection t_f after the beginning of an antiblock. Consequently, the last vehicle can enter the intersection $t_c - t_f$ before the arrival of the next major stream arrival. In terms of a queuing process, the entry time is the time of departure from service. In Figure 2.11 major stream arrival times can be interpreted as service departure times, while minor stream entry times should be considered as service entry times. See Figure 3.13 on page 61 for more details.

The operation can be described as a *single server queuing system* with deterministic (t_f) service times. Customers can enter service during antiblock intervals only. A more comprehensive description of several conflicting streams would be a single server system with *priority queues*. Each customer type (rank) has a separate queue. The first customer in the highest rank queue is always served first. Switching from one queue to another requires a “reorientation time” $t_c - t_f$ (Jaiswal 1968). The hierarchy of conflicts in an entire unsignalized intersection is, however, too complicated to be described by a queuing system. For example, in a four-legged intersection (Fig. 2.2) streams 4 and 8 have a conflict, as do streams 3 and 4, but stream 3 and 8 vehicles can enter the intersection simultaneously. Queuing models are very limited, but nevertheless useful tools in the analysis of unsignalized intersections.

It is assumed that service starts as a vehicle crosses stop line. Service time is equal to the follow-up time. When service ends, the next vehicle can enter service (cross stop line).

The reference point of a priority stream is the conflict point between the priority and minor streams. However, in the gap acceptance process no service times are assumed for priority streams. The entry of minor stream vehicles is blocked by critical gaps only. Follow-up times are applied to minor stream vehicles.

In Figure 2.11 the first vehicle enters the intersection at the same time as the major stream vehicle arrives ($\delta_1 = \tau_3$ and $\delta_4 = \tau_5$); i.e., passes the conflict point. When the “service is completed”, the minor stream vehicle passes the conflict point. Thus, t_f is also the minimum time interval between a major stream vehicle and a merged minor stream vehicle. This is, however, an extension of the basic queuing model.

The service of a minor stream vehicle may overlap the critical gap. In Figure 2.11 vehicle 3 departs service at $\delta_3 + t_f$, when major stream vehicle 4 has already blocked the entry of minor stream vehicle 4. The minimum headway between a minor stream vehicle and a major stream vehicle is $t_c - t_f$.

This discussion raises three issues concerning time intervals between vehicles.

1. Critical gap (t_c) cannot be shorter than the follow-up time (t_f). Otherwise the arrival time of the next major stream vehicle (such as τ_6 in Figure 2.11) might take place before a minor stream vehicle has departed from service ($\delta_5 + t_f$).
2. Even if $t_c > t_f$ the headway between the merged vehicle and the next major stream vehicle ($\tau_6 - \delta_5 - t_f$) may become too short, in which case the major stream vehicle has to decelerate in order to delay its arrival and to keep the headway safe. Because the priority of major stream vehicles is limited, this type of operation is called “limited priority merge” (see Troutbeck 2002). Major stream priority becomes limited, if $t_c < t_f + t_p$, where t_p is the follower headway between major stream vehicles.—To make the argument simpler, follower headway (headway between vehicles in a platoon) is assumed constant.
3. The minor stream follow-up time (t_f) cannot be shorter than the major

stream follower headway (t_p). Otherwise the minor stream queue discharge rate would be higher than the major stream capacity. The major stream cannot receive merging vehicles at this rate.

Because drivers and vehicles differ, the minimum gaps and lags accepted by drivers and the headways between discharging vehicles differ from vehicle to vehicle. Vehicle behavior is heterogeneous. In addition, even under similar conditions a driver may behave differently at different times. A driver may accept a gap that is shorter than a gap rejected by the same driver earlier. This kind of behavior is called inconsistent. The critical gap of impatient drivers become shorter as waiting time increases.

In mathematical models driver behavior is usually assumed to be both consistent and homogeneous. A *consistent* driver behaves in the same way every time at similar situations. If the behavior has a stochastic element, it follows the same statistical distribution. If drivers are *homogeneous*, there is no difference in behavior among different drivers under similar conditions. Possible stochastic elements follow same distributions among all drivers.

The assumption of consistent and homogeneous gap acceptance behavior is clearly unrealistic. It has been suggested that capacity would be increased if drivers were heterogeneous and decreased if drivers were inconsistent. The overall effect of consistent and homogeneous drivers is minimal (Catchpole and Plank 1986, Troutbeck 1988, Transportation Research Board 1997).

In summary, the major assumptions in basic gap acceptance models are

1. Similar (consistent) behavior of each driver under similar conditions
2. Similar (homogeneous) behavior among all drivers
3. Constant critical gaps and follow-up times under similar conditions and critical lags equal to critical gaps
4. Same critical gaps in all priority streams and between vehicles of different priority streams
5. Gaps of all independent priority streams are combined into a single stream of gaps
6. The availability of priority-stream gaps is described by a statistical headway distribution model
7. Minor-road vehicles arrive randomly
8. Priority stream vehicles do not adjust their behavior to accommodate an entering vehicle; i.e., gap-acceptance is similar at all traffic volumes
9. Minor stream vehicles can distinguish priority vehicles from vehicles exiting before the conflict area

It is also assumed that follow-up time is shorter than critical gap. This assures that under a continuous queue there is always a vehicle waiting for entry as a new gap begins. If critical gap was shorter than follow-up time, the mathematical models would allow the possibility of queue discharge headways shorter than follow-up times.

Several models have relaxed one or more of these assumptions. Ashworth (1969) and Plank and Catchpole (1984) have developed a capacity formula for heterogeneous and consistent drivers. Catchpole and Plank (1986) and Madanat, Cassidy and Wang (1994) have analyzed the operation of unsignalized intersections for heterogeneous and inconsistent drivers; thus relaxing assumptions 1, 2, and 3. Troutbeck has relaxed assumption 8 and studied gap acceptance under "limited priority merge" (Bunker and Troutbeck 1994*b*, Bunker and Troutbeck 1994*a*, Troutbeck 1995, Troutbeck 1997, Troutbeck and Kako 1999, Troutbeck 1999).

Tanner (1967) was the first to relax assumption 5. He estimated minor-stream capacity assuming two or more independent priority streams with different flow rates. See also Troutbeck (1986). Gazis, Newell, Warren and Weiss (1967) and Fisk (1989) have extended the model to include the case where different lanes (priority streams) have different critical gaps (assumption 4).—Only streams without mutual conflicts can be considered independent. When priority streams are from more than one Rank special analysis methods are required.

The effect of nonpriority vehicles, such as major road right turners, can be taken into a model by adding a proportion of this stream to a priority stream. In advanced methodologies many of these assumptions have been relaxed, but in the core of these methods is usually a potential capacity formula with most of these assumptions. Some adjustments are then presented to make the method more general.

The nature of a problem often becomes clearer when it is compared to other, similar problems. In a mathematical sense the following traffic engineering problems have many similarities with the unsignalized intersection case:

The gap acceptance model in an unsignalized intersection is similar to the problem of *pedestrians crossing a road*. Haight (1963) refers to the items delayed as vehicles or pedestrians "according as they do or do not form queues". Early studies of this problem have been presented by Adams (1936), Tanner (1951) and others. If all waiting pedestrians can cross the road when a headway larger than the critical gap starts, the follow-up time is null. This makes the analysis simpler. However, if all waiting pedestrians can cross the road during a headway larger than the critical gap, the capacity of a pedestrian crossing is either infinite or null. The model makes more sense in delay studies than in capacity studies.—Detailed information of pedestrian gap acceptance and vehicle headway distributions can be used to evaluate the accident risk of pedestrians (Cohen, Dearnaley and Hansel 1955, Song, Dunne and Black 1993, Pasanen 1991).

Tanner (1951) remarked the similarity between the pedestrian crossing problem and the operation of vehicle-actuated *traffic signals* as described by Garwood (1940). Beckmann et al. (1956) presented the operation of a stop-controlled intersection in terms of red (block) and green (antiblock) interval sequences generated

by the priority stream gaps. A minor approach has “green” when conflicting major streams have no “green demand”. The generation and extension of blocks (Fig. 2.11) is mathematically very similar to the operation of vehicle detectors (Fig. 2.12). In the theory of stochastic processes this type of operation is known as a Type II counter (Parzen 1962). Akçelik (1994) has described a traffic-signal analogy in modeling gap-acceptance at unsignalized intersections.

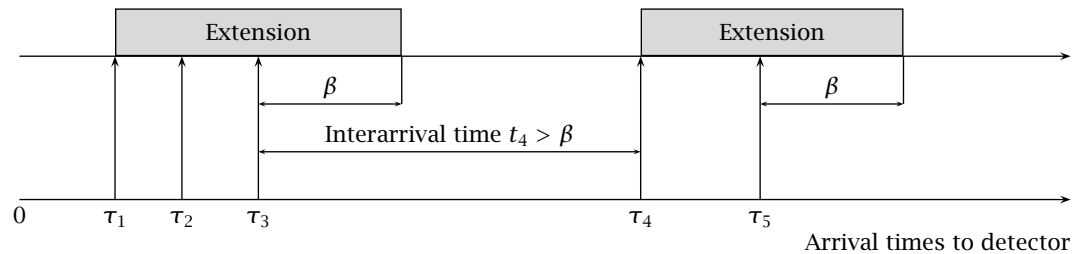


FIGURE 2.12 · Extension principle of vehicle detectors

Another model similar to the unsignalized intersection is the operation of *merging zones* (see Haight, Bisbee and Wojcik 1962, Haight 1963, Drew 1967). Contrary to the minor road drivers at unsignalized intersections merging drivers can change the gap acceptance criterion by changing their speeds, and they have a limited opportunity to determine the location of the merge. In more advanced models the effect of turbulence on priority streams should be considered (Transportation Research Board 2000).

Passing on two-lane highways is a gap-acceptance process, where a passing vehicle moves in the opposite direction to the “priority stream”. Drivers can determine the time and location of passing considering gap availability and sight restrictions. The order in which vehicles pass each others is not fixed, and several vehicles on a road segment may pass simultaneously. Consequently, this problem is usually described at a macroscopic level. For an overview of these models see McLean (1989) and Luttinen (2001). Miller and Pretty (1968) have discussed the gap-acceptance of inconsistent drivers.

The discussion above gives a toolbox to be used in the development of a capacity analysis theory for unsignalized intersections.

3

Capacity models

3.1 Capacity of Rank 2 streams

3.1.1 Potential capacity under one priority stream

The theoretical foundation for capacity analysis of unsignalized intersections was laid in the 1960's by J. C. Tanner (1962) and Major and Buckley (1962). Consequently, the first edition of HCM (Bureau of Public Roads 1950) did not discuss unsignalized intersections. Even the second edition (Highway Research Board 1965) did not present any analysis methodology for unsignalized intersections. Two tables were presented as examples of capacities of "four-way stop intersections". By the time of the third edition (Transportation Research Board 1985) theoretical models and empirical field data were at such a level that enabled the presentation of specific analysis procedures. The HCM methodology was adopted from German guidelines (Forschungsgesellschaft für das Straßenwesen 1972), which were based on the research by Harders (1968). See Baass (1987) and Brilon (1988a).

Capacity is the hourly rate at which vehicles can enter the intersection during a given time period under prevailing conditions. Capacity is estimated for streams of ranks lower than 1. The methods described below cannot be applied to uncontrolled intersections or Rank 1 flows. The German manual (Forschungsgesellschaft für Straßen- und Verkehrswesen 2001) suggest 1,800 pc/h as a maximum flow rate for Rank 1 through streams. At uncontrolled intersections the manual estimates the maximum number of vehicles entering the intersection from all four approaches to be in the range 600–800 pc/h.

This section describes the basic theoretical formulas used to estimate the *potential capacity* of unsignalized intersections. Only a simple crossing operation, as in Figure 2.1, or a simple merge operation, as in a roundabout entry (Fig. 2.4), is discussed. The road and traffic conditions are assumed optimal. Unless stated otherwise, the models follow the assumptions presented above (page 37).

Potential capacity describes the capacity of a minor stream under ideal conditions assuming that it is unimpeded by other movements and has exclusive use

of a separate lane (Transportation Research Board 2000). When the impedance effects (queuing in higher priority streams) are considered, HCM2000 calls the capacity estimate a *movement capacity*. Because Rank 2 vehicles do not face any impedance effects due to queuing at higher priority streams, the movement capacity of a Rank 2 stream is equal to the potential capacity:

$$C_{m,k} = C_{p,k}, \quad k \in \{1, 4, 9, 12\}. \quad (3.1)$$

It is assumed that there is a continuous queue of minor stream vehicles (stream 11 in Fig. 2.1) waiting to enter the intersection. The expected number of vehicles departing from the queue during a headway t is described by a *gap acceptance function*

$$\alpha(t) = \sum_{i=0}^{\infty} i p_i(t) \quad (3.2)$$

where $p_i(t)$ is the probability of i departures during headway t .

The expected number of departing vehicles during a randomly selected headway is obtained by integration

$$\mathbb{E}[\alpha] = \int_0^{\infty} \alpha(t) f(t) dt, \quad (3.3)$$

where $f(t)$ is the probability density function of major stream headways. Potential capacity is obtained by multiplying $\mathbb{E}[\alpha]$ by the number of major stream headways per time unit (q_M) (Siegloch 1973)

$$C_p = q_M \mathbb{E}[\alpha] = q_M \int_0^{\infty} \alpha(t) f(t) dt. \quad (3.4)$$

Accordingly, capacity is expressed in terms of the gap-acceptance function $\alpha(t)$ and the headway distribution.

A minor stream vehicle enters the intersection, if there is a headway or lag larger or equal to the critical gap t_c in the priority stream. Critical lag is assumed equal to the critical gap. Vehicles entering the intersection during the same priority stream headway have discharge headways equal to the follow-up time t_f .

During a headway of $0 < t < t_c$ no vehicles can enter the intersection. One vehicle can enter during a headway of $t_c \leq t < t_c + t_f$. Two vehicles can enter during a headway $t_c + t_f \leq t < t_c + 2t_f$. Accordingly

$$p_i(t) = \begin{cases} 1, & \text{if } t_c + (i-1)t_f \leq t < t_c + it_f \\ 0, & \text{otherwise.} \end{cases} \quad (3.5)$$

Gap acceptance function is a step function

$$\alpha(t) = \left\lfloor \frac{t - t_c + t_f}{t_f} \right\rfloor_+ = \begin{cases} 0, & \text{if } t < t_c \\ \left\lfloor \frac{t - t_c + t_f}{t_f} \right\rfloor, & \text{if } t \geq t_c, \end{cases} \quad (3.6)$$

where $\lfloor X \rfloor$ is the greatest integer not larger than X (floor function).

The probability of i arrivals during a randomly selected headway is the probability that the headway has an appropriate length:

$$\begin{aligned}
 \mathbb{P}\{\alpha = i\} &= \mathbb{P}\{t_c + (i-1)t_f \leq t < t_c + it_f\} \\
 &= \int_{t_c + (i-1)t_f}^{t_c + it_f} f(u) du \\
 &= F(t_c + it_f) - F(t_c + (i-1)t_f) \\
 &= R(t_c + (i-1)t_f) - R(t_c + it_f),
 \end{aligned} \tag{3.7}$$

where $f(t)$ is the probability density function, $F(t)$ is the cumulative distribution function, and $R(t) = 1 - F(t)$ is the survival function of the major road headway distribution.

Capacity (maximum number of entering vehicles per time unit) is obtained by multiplying the expected number of minor stream vehicles entering during a major stream headway by the expected number of major road headways (vehicles) per time unit:

$$\begin{aligned}
 C_p &= q_M \sum_{i=0}^{\infty} i \mathbb{P}\{t_c + (i-1)t_f \leq t < t_c + it_f\} \\
 &= q_M \sum_{i=0}^{\infty} i \int_{t_c + (i-1)t_f}^{t_c + it_f} f(u) du \\
 &= q_M \sum_{i=0}^{\infty} i [R(t_c + (i-1)t_f) - R(t_c + it_f)] \\
 &= q_M \sum_{i=0}^{\infty} R(t_c + it_f) \quad (\text{veh/h}).
 \end{aligned} \tag{3.8}$$

Capacity is high, if the frequency of large headways is high. Consequently, capacity increases as the coefficient of variation of priority stream headways increases (Luttinen 1990). If priority stream flow rate is null, the minor road queue discharges at headways t_f , and the capacity is t_f^{-1} veh/s or $3600t_f^{-1}$ veh/h.

If major stream headways follow the *negative exponential distribution*, the minor stream capacity is (Major and Buckley 1962, Drew 1968, Harders 1968)¹

$$\begin{aligned}
 C_p &= q_M \sum_{i=0}^{\infty} R(t_c + it_f) \\
 &= q_M \sum_{i=0}^{\infty} e^{-\lambda(t_c + it_f)} \\
 &= q_M e^{-\lambda t_c} \sum_{i=0}^{\infty} e^{-i\lambda t_f} \\
 &= \frac{q_M e^{-\lambda t_c}}{1 - e^{-\lambda t_f}} \\
 &= \boxed{\frac{q_M e^{-q_M t_c / 3600}}{1 - e^{-q_M t_f / 3600}}}, \quad 0 \leq t_f \leq t_c.
 \end{aligned} \tag{3.9}$$

¹According to Major and Buckley (1962) this equation was first obtained by Fisher (1960).

See Figure 3.1. The result was obtained using the convergence property of a geometric series; i.e.,

$$\sum_{i=0}^{\infty} a^i = \frac{1}{1-a}, \quad a < 1. \quad (3.10)$$

This capacity equation (3.9) is used in the 1985 HCM (Transportation Research Board 1985, Baass 1987) and HCM2000 (Transportation Research Board 2000) as well as in the Danish DanKap (Vejdirektoratet 1999) and in the Norwegian guidelines (Statens Vegvesen 1985, Aakre 1998). The Swedish Capcal 2 uses this equation for minor through and left turn movements, and when the priority streams occupy at least four lanes (Swedish National Road Administration 1995, Hagring 1997).

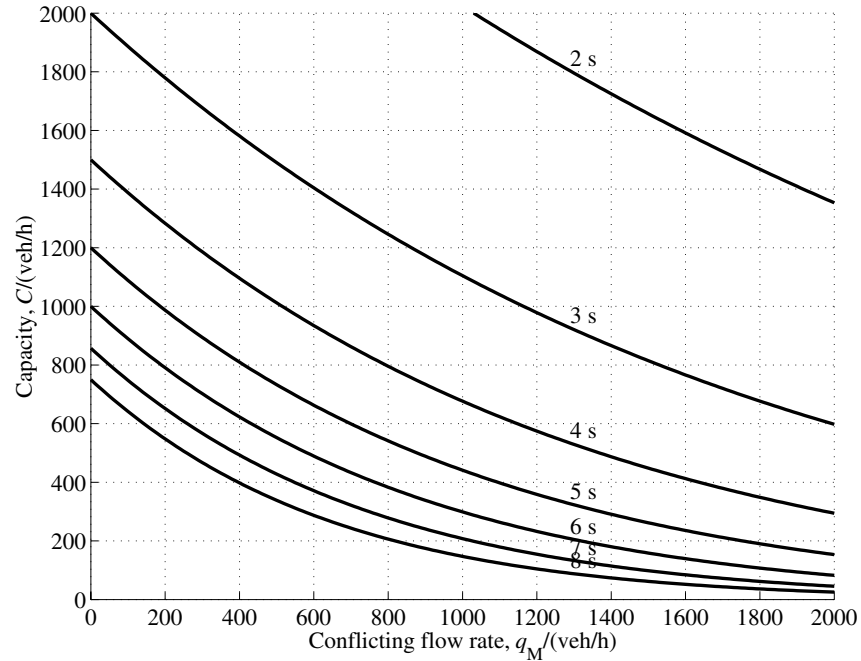


FIGURE 3.1 · Minor-stream potential capacity with different critical gaps, $t_f = 0.6t_c$, and major-stream headways following the negative exponential distribution

If follow-up time is null, the entire queue can discharge simultaneously, and the capacity reaches infinity. This is the capacity of a simple pedestrian crossing model.

For priority stream headways following the *shifted exponential distribution* the

minor stream potential capacity can be obtained as (Luttinen 1990)

$$\begin{aligned}
 C_p &= q_M \sum_{i=0}^{\infty} R(t_c + it_f) \\
 &= q_M \sum_{i=0}^{\infty} e^{-\theta(t_c + it_f - t_p)} \\
 &= q_M e^{-\theta(t_c - t_p)} \sum_{i=0}^{\infty} e^{-i\theta t_f} \\
 &= \frac{q_M e^{-\theta(t_c - t_p)}}{1 - e^{-\theta t_f}}, \quad 0 \leq t_p \leq t_c, 0 \leq t_f \leq t_c.
 \end{aligned} \tag{3.11}$$

If $t_p = 0$ we get the potential capacity for exponential headways (3.9). When substitution (2.21) for θ is made, capacity can be expressed as

$$C_p = \frac{q_M \exp\left(\frac{-q_M(t_c - t_p)}{3600 - q_M t_p}\right)}{1 - \exp\left(\frac{-q_M t_f}{3600 - q_M t_p}\right)} \quad (\text{veh/h}). \tag{3.12}$$

As Figure 3.2 displays minor stream potential capacity decreases as the major stream minimum headway (t_p) increases and variance decreases (see Luttinen 1990, Luttinen 1996). The loss in capacity is largest at high major stream flow rates.

The survival function (2.29) of *Cowan's M3 distribution* differs from the shifted exponential survival function (2.18) by the factor ϕ . Thus, the potential capacity is (Plank 1982):

$$\begin{aligned}
 C_p &= q_M \sum_{i=0}^{\infty} R(t_c + it_f) \\
 &= q_M \phi \sum_{i=0}^{\infty} e^{-\gamma(t_c + it_f - t_p)} \\
 &= \frac{q_M \phi e^{-\gamma(t_c - t_p)}}{1 - e^{-\gamma t_f}}, \quad 0 \leq t_p \leq t_c, 0 \leq t_f \leq t_c, 0 \leq \phi \leq 1.
 \end{aligned} \tag{3.13}$$

It can be interpreted as minor stream capacity across a stream, where the proportion ϕ of headways are available for gap acceptance and these headways follow the shifted exponential distribution. This is a special case of the capacity equation of Tanner (1967). If $\phi = 1$ and $t_p = t_p$, we get the capacity for shifted exponential headways (3.11). For exponential headways (3.9) we set $\phi = 1$ and $t_p = 0$.

In the Australian roundabout capacity guideline (Austroads 1993) parameter γ has been defined following the Tanner headway model (2.40) as

$$\gamma = \frac{\phi q_M}{3600 - q_M t_p}, \tag{3.14}$$

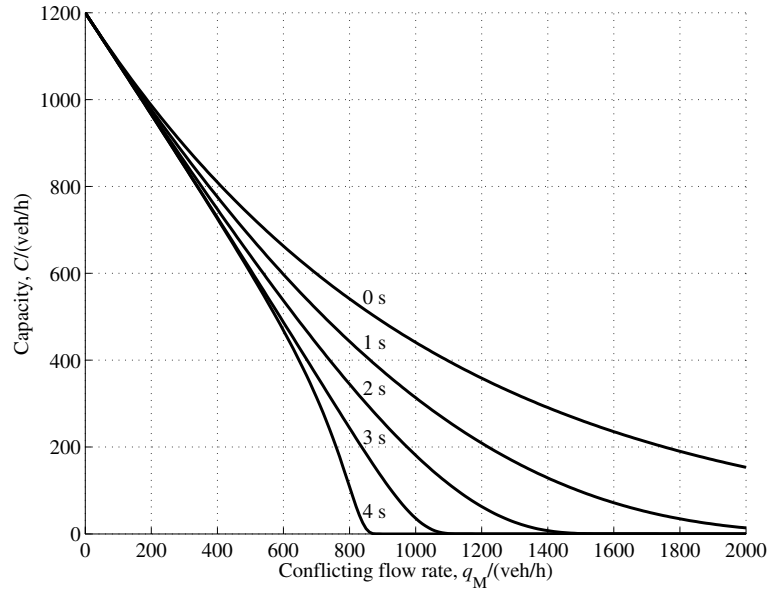


FIGURE 3.2 · Minor stream potential capacity when critical gap is 5 s, follow-up time 3 s, and major stream headways follow shifted exponential distribution with minimum headway (location parameter) ranging from zero to four seconds

where $q_M = q_c$ is the circulating flow rate at the entry. The proportion of free headways is, however, lower than predicted by Tanners M/D/1 model:

$$\phi = 0.75(1 - \lambda t_p) = 0.75 \left(1 - \frac{q_M t_p}{3600} \right). \quad (3.15)$$

See also Troutbeck (1993) and Troutbeck and Kako (1997).

Troutbeck and Kako (1997) have modified capacity equation (3.13) for “limited priority merge”. It describes the reduced capacity due to major stream vehicles slowing down in order to allow enough space for a merging minor-stream vehicle. The delay of a major-stream vehicle reduces the next gap. On the other hand, shorter critical gaps (t_c) during limited priority operation increase capacity.

Some major-stream vehicles have to slow down, if critical gap t_c is less than $t_f + t_p$. These vehicles have to delay their entry in order to keep the headway to a merged vehicle at t_p (see Figure 2.11 and related discussion above). The capacity of limited priority merge is

$$C_L = \frac{q_M f_L \phi e^{-\gamma(t_c - t_p)}}{1 - e^{-\gamma t_f}}, \quad t_p < t_c < t_f + t_p, \quad (3.16)$$

where

$$f_L = \frac{1 - e^{-\gamma t_f}}{1 - e^{-\gamma(t_c - t_p)} - \gamma(t_c - t_p - t_f) e^{-\gamma(t_c - t_p)}} \quad (3.17)$$

is the adjustment factor for limited priority merge (Fig. 3.3). Parameter γ is calculated as in equation (3.14). If $t_c > t_f + t_p$, major stream vehicles have absolute priority and f_L should be set to 1. See also Bunker and Troutbeck (1994b), Bunker and Troutbeck (1994a), Troutbeck (1995), Troutbeck (1997), and Troutbeck and Kako (1999).

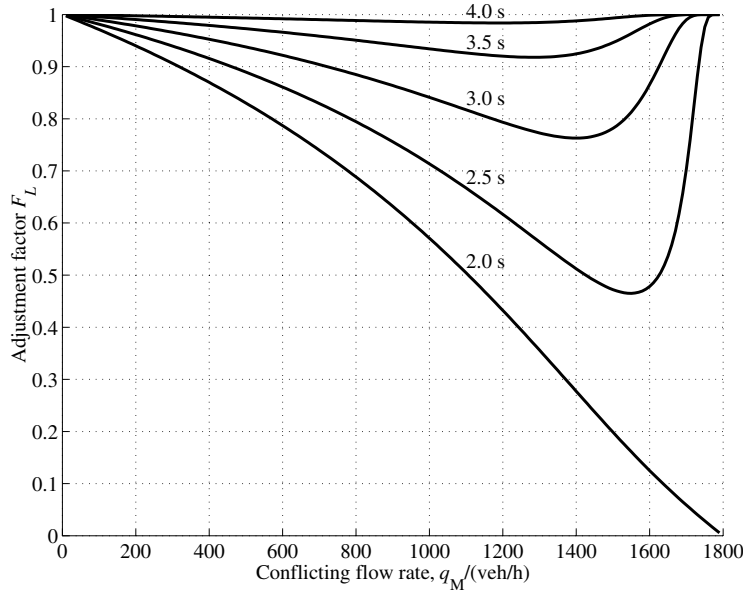


FIGURE 3.3 · Adjustment factor f_L with $t_f = 2.5$ s, $\phi = 0.8$, and $t_p = 2$ s (Troutbeck 1995)

If the traffic flow is considered as departures from a M/D/1 queuing system, substitutions (2.39) and (2.40) are made, and we obtain the capacity formula presented by Tanner (1962)

$$C_p = \frac{q_M (1 - \lambda t_p) e^{-\lambda(t_c - t_p)}}{1 - e^{-\lambda t_f}}, \quad (3.18)$$

where $\lambda = q_M/3600$ veh/s. (Tanner obtained the equation as a limiting case when delay approaches infinity.) Tanner's formula has since been used in numerous manuals (e.g. Road Research Laboratory 1965, Hobbs 1974, Austroads 1988). Capcal2 uses this model for major road left turns and minor road right turns when the major flow occupies one lane only (Swedish National Road Administration 1995, Hagring 1997).

Figure 3.4 displays minor stream capacity according to Tanner (1962) with different critical gaps and $t_f = 0.6t_c$. The follower headway in a platoon is $t_p = 1.8$ s, as in Capcal 2 (Swedish National Road Administration 1995, Hagring 1997). The platooning model decreases the frequency of large headways at high flow rates more steeply than the exponential headway model (Fig. 3.1). When all

vehicles travel in platoons, $q_M = 3600/1.8 = 2000$ veh/h and there is no capacity left to a minor stream.

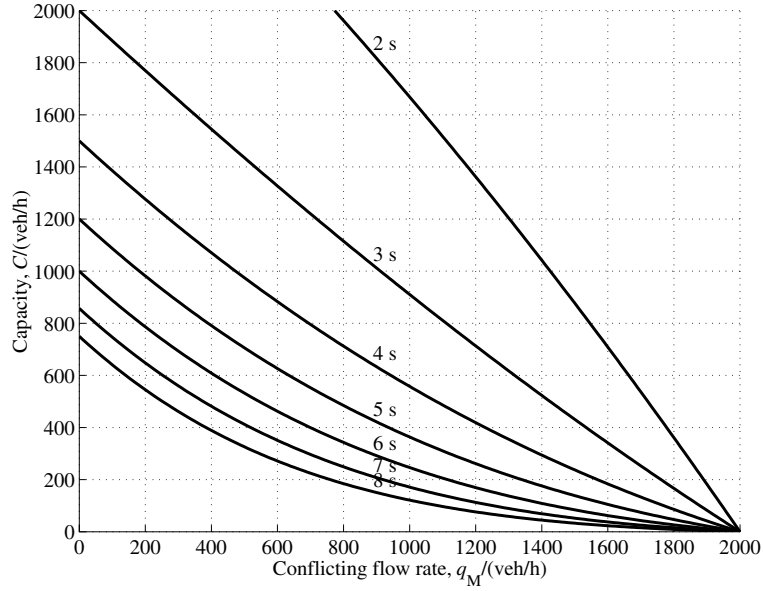


FIGURE 3.4 · Capacity according to Tanner with different critical gaps, $t_f = 0.6t_c$, and platoon headways $t_p = 1.8$ s

Figure 3.5 shows the effect of major stream bunching on minor stream capacity. As bunching increases (ϕ decreases) the proportion of short intra-bunch headways increases, and so does also the average length of free headways. This improves capacity. If $\phi = 1.0$, there is no bunching, and the major stream headways follow the shifted exponential distribution. Shifted exponential distribution has a lower coefficient of variance than the negative exponential distribution and gives lower capacity for minor stream vehicles. This is, however, just a feature of the M3 model, and it cannot be taken as an evidence of capacity reduction at low platoon percentages. By more extensive comparisons Siegloch (1973) concluded that slight disturbances (bunching) reduce capacity, medium disturbances increase capacity slightly, and severe disturbances increase capacity significantly (Brilon 1988a).

A comparison of figures 3.1 and 3.4 indicates that at high major stream flow rates, when the variation of headways decreases, the minor stream capacity (3.13) decreases below the capacity of the exponential model (3.9). This could be expected by the shape of the coefficient of variation curve (Fig. 2.9).

Figure 3.6 displays the capacity a minor stream under limited priority conditions based on Tanner's headway (M/D/1 output) distribution with flow-dependent critical gaps ($t_c = 5.0 - 6.0\lambda$) s and follow-up times ($t_f = 0.6t_c$) s and constant ($t_p = 1.8$ s) follower headway in platoons. It is compared with Tanner's capacity formula (absolute priority) with the same critical gaps, follow-up

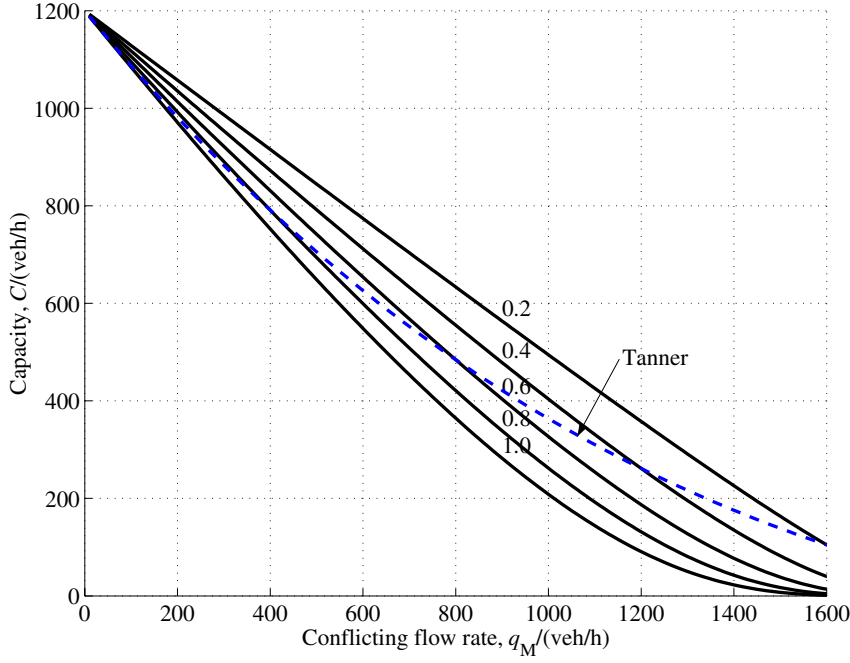


FIGURE 3.5 · Capacity with different proportions of free vehicles, critical gap $t_c = 5$ s, follow-up time $t_f = 3$ s, and intra-platoon headway $t_p = 1.8$ s

times, and follower headways. Under flow dependent gap acceptance the capacity curve has steepest slopes at low and high flow rates. This can be inferred from Tanner's capacity formula (Fig. 3.4) by assuming large (5 s) critical gaps at low flow rates and small (2 s) critical gaps at high flow rates. If critical gaps decrease with increasing flow rate the capacity curve becomes more linear, especially under limited priority. This gives some theoretical support to the British linear regression models (Kimber and Coombe 1980).

In a *fluid analogy model* gap acceptance function is continuous for headways larger than a threshold t_0 . A *linear gap acceptance function* has been suggested by Siegloch (1973):

$$\alpha(t) = \left(\frac{t - t_0}{t_f} \right)_+ = \left(\frac{t - t_c + \frac{t_f}{2}}{t_f} \right)_+, \quad (3.19)$$

where $t_0 = t_c - t_f/2$ is the shortest acceptable headway. The minor stream departure process is considered as a fluid flowing into the headways of the priority stream. As long as the lag is larger than t_0 the minor stream departure flow rate is t_f^{-1} . See Figure 3.7.

Equation (3.19) is a linear approximation of the step function (3.6). Linear gap acceptance function is a simplified means to describe the average gap acceptance

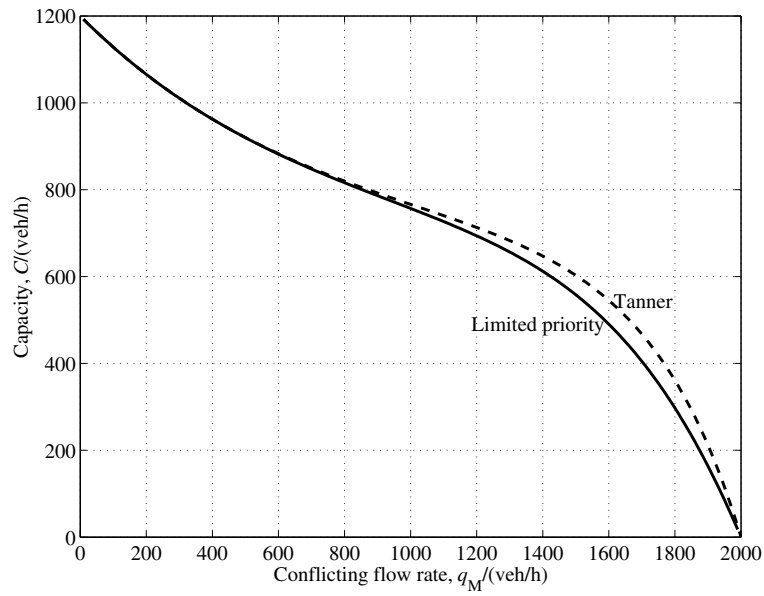


FIGURE 3.6 · An example of minor stream capacity under flow dependent gap acceptance and major stream arrivals from an M/D/1 queuing system with limited (solid curve) or absolute (dashed curve) priority

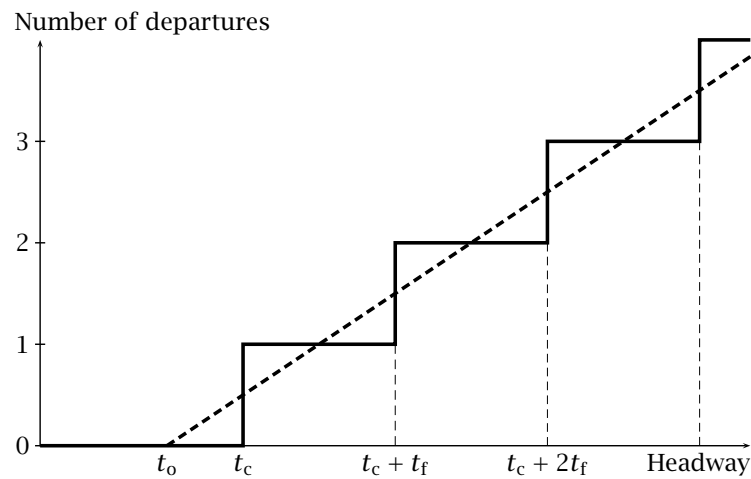


FIGURE 3.7 · Stepwise and linear gap acceptance functions

behaviour of heterogeneous drivers. The potential capacity is obtained as

$$\begin{aligned}
 C_p &= q_M \int_0^\infty \alpha(t) f(t) dt \\
 &= q_M \int_{t_o}^\infty \frac{t - t_o}{t_f} f(t) dt \\
 &= \frac{q_M}{t_f} \left[\int_{t_o}^\infty t f(t) dt - t_o R(t_o) \right].
 \end{aligned} \tag{3.20}$$

For negative exponential priority stream headways and linear gap acceptance function the potential capacity is obtained as (Siegloch 1973, Schnabel and Lohse 1997)

$$\begin{aligned}
 C_p &= q_M \int_{t_o}^\infty \frac{t - t_o}{t_f} \lambda e^{-\lambda t} dt \\
 &= \frac{\lambda q_M}{t_f} \left[\int_{t_o}^\infty t e^{-\lambda t} dt - t_o \int_{t_o}^\infty e^{-\lambda t} dt \right] \\
 &= \frac{q_M}{t_f} \left[(\lambda^{-1} - t - t_o) e^{-\lambda t} \right]_{t=t_o}^\infty \\
 &= \frac{q_M}{\lambda t_f} e^{-\lambda t_o} \\
 &= \boxed{\frac{3600}{t_f} e^{-q_M t_o / 3600}}.
 \end{aligned} \tag{3.21}$$

Because t_f is the minimum headway and the exponential part is the probability (proportion of time) of a lag larger than t_o , the equation can be expressed as

$$\boxed{C_p = q_{\max} \mathbb{P}\{Y > t_o\} = q_{\max} R_Y(t_o)}, \tag{3.22}$$

where $q_{\max}$ is the maximum flow rate of the minor stream in the case of no higher priority flow. $\mathbb{P}\{Y > t_o\}$ is the proportion of time that minor stream vehicles find an acceptable lag. During this time minor stream vehicles can discharge at maximum rate. In a fluid analogy model the potential capacity can be expressed as the *product of maximum minor stream flow rate ($q_{\max}$) and the probability of a lag larger than the minimum acceptable gap (t_o)*.

In the German guidelines (Brilon et al. 1994, Forschungsgesellschaft für Straßen- und Verkehrswesen 2001) equation (3.21) has been used, but the difference between equations (3.21) and (3.9) has been considered insignificant (see Brilon 1991). The comparison of Figures 3.1 and 3.8 demonstrates that the capacities of equations (3.9) and (3.21) are virtually identical. According to Brilon and Stuwe (1993) Siegloch's equation (3.21) with parameters estimated by regression analysis was slightly better as a capacity estimate for German roundabouts than a linear regression equation. — HCM has applied equation (3.21) in the 1994 update (Transportation Research Board 1994).

Equation (3.21) has a simpler form than equation (3.9). It can be transformed into a linear equation by taking a logarithm. This enables parameter estimation

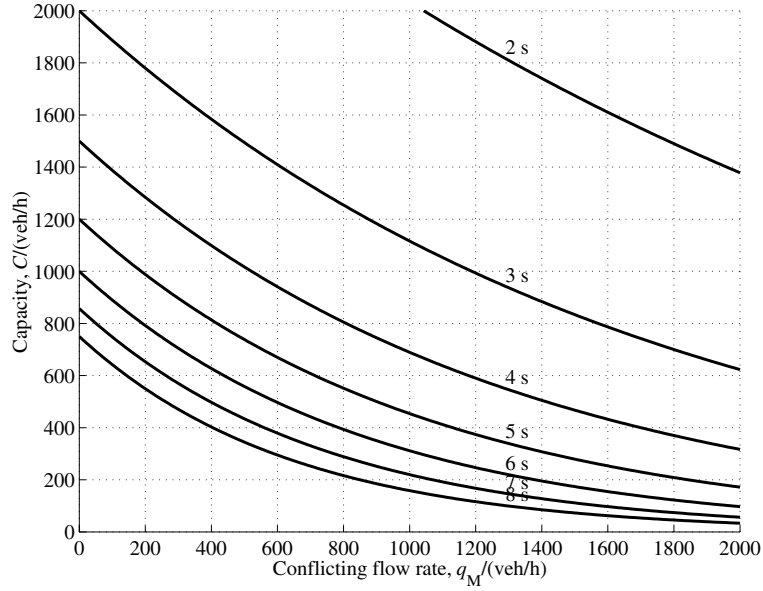


FIGURE 3.8 · Minor-stream capacity with different critical gaps, $t_f = 0.6t_c$, linear gap acceptance function and major stream headways following the negative exponential distribution

by linear regression analysis (Guichet 1997, Aagaard 1995). The model also has a simple logic: During the time available for gap acceptance minor stream discharges at maximum rate. The operation of individual vehicles can be ignored.

For shifted exponential headways the potential capacity is

$$\begin{aligned}
 C_p &= \frac{q_M}{t_f} \left[\int_{t_o}^{\infty} t \theta e^{-\theta(t-t_p)} dt - t_o R(t_o) \right] \\
 &= \frac{q_M}{t_f} \left[\left(t_o + \frac{1}{\theta} \right) e^{-\theta(t_o-t_p)} - t_o e^{-\theta(t_o-t_p)} \right] \\
 &= \frac{q_M}{\theta t_f} e^{-\theta(t_o-t_p)}, \quad t_o > t_p.
 \end{aligned} \tag{3.23}$$

By using the substitution of equation (2.21) for θ , the capacity can be expressed as

$$C_p = \frac{3600 - q_M t_p}{t_f} e^{-\theta(t_o-t_p)}, \quad t_o > t_p. \tag{3.24}$$

The equation can also be expressed in the familiar form

$$C_p = \frac{3600 e^{-\theta(t_o-t_p)}}{t_f(1 + \theta t_p)} = q_{\max} \mathbb{P}\{Y > t_o\}, \quad t_o > t_p. \tag{3.25}$$

Because the smallest acceptable headway is larger than the follower headway in a platoon ($t_o > t_p$), in Cowan's M3 distribution only the exponential tail of the

distribution has an effect on the capacity. It differs from the shifted exponential distribution by factor ϕ , thus giving the following potential capacity equation (Transportation Research Board 1997)

$$\begin{aligned} C_p &= \frac{q_M}{t_f} \left[\phi \gamma \int_{t_o}^{\infty} t e^{-\gamma(t-t_p)} dt - t_o R(t_o) \right] \\ &= \frac{\phi q_M}{\gamma t_f} e^{-\gamma(t_o-t_p)}, \quad t_o > t_p. \end{aligned} \quad (3.26)$$

Equation (2.40) indicates that for an M/D/1 departure process $\phi \gamma^{-1} = \bar{t} - t_p = \lambda^{-1} - t_p$. When this substitution and substitution for $q_M = 3600\lambda$ are made, the modification of Tanner's capacity model (3.18) is obtained as:

$$C_p = \frac{3600(1 - \lambda t_p)}{t_f} e^{-\lambda(t_o-t_p)}, \quad t_o > t_p, \quad (3.27)$$

where parameter $\lambda = q_M/3600$ veh/s (Brilon 1988a, Transportation Research Board 1997). This equation can also be presented as

$$C_p = \frac{3600\phi e^{-\gamma(t_o-t_p)}}{t_f(\phi + \gamma t_p)} = q_{\max} \mathbb{P}\{Y > t_o\}, \quad t_o > t_p. \quad (3.28)$$

The results are virtually identical with the original Tanner model (3.18) as displayed in Figure 3.4.

3.1.2 Potential capacity under multiple priority streams

The potential capacity models described above assume a single conflicting priority (major) stream on one lane. In most cases a minor stream faces many priority streams, some of which may share lanes with other streams or occupy several lanes. This section discusses the effect of multiple *independent* priority streams. Sections 3.2 and 3.3 discuss the capacity under dependent priority streams. The effect of shared lanes is discussed in section 3.4.

Let us consider a T-intersection (Fig. 2.3 on page 21) with an exclusive left-turn lane in the major road. The major road through movements (2 and 5) can be considered as two independent priority streams for the minor road left turn stream (7). The set of independent priority streams for minor stream 7 is $I_7 = \{2, 5\}$. Stream 4 (major road left turn) is not independent, because it must give way to streams 2 and 3.

On multi-lane highways (Fig. 3.9) traffic flows on adjacent through and right turn lanes can be considered independent. Minor road right turning vehicles (stream 9) must give way to traffic flow on one major road lane; i.e., $I_9 = \{2a\}$. Minor road left-turn vehicles (7) must give way to other major road through streams, but they can ignore stream 5a; i.e., $I_7 = \{2a, 2b, 5b\}$.

If the independent priority streams are described as point processes, the combined priority stream is a *superposition* of the component point processes.² As

²In reliability theory a superimposed process is a system having several components in series. When a component fails it is immediately replaced with a new component of the same type. (Thompson 1988, Høyland and Rausand 1994)

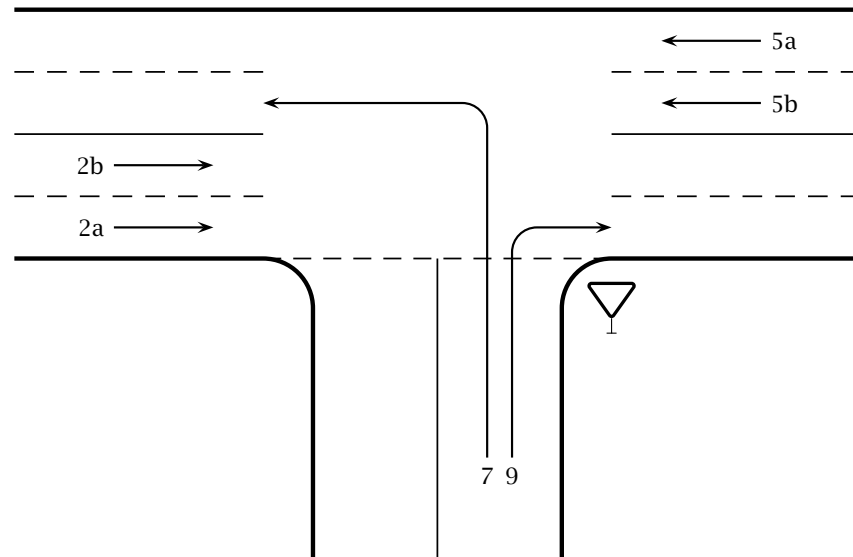


FIGURE 3.9 · T-intersection of a four-lane major road

Figure 3.10 indicates, the superimposed process does not have the minimum headway restrictions that the component processes may have.

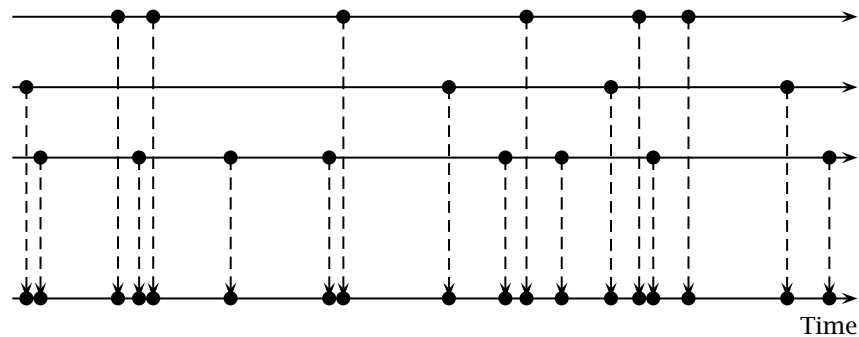


FIGURE 3.10 · Superposition of three independent point processes

A superimposed process is more like a Poisson process than the component processes (Cox and Isham 1980). Palm (1943) proposed that the superposition of a large number of independent sparse point processes leads to a Poisson process in the limit. This first limit theorem of point processes was proved by Khintchine (1960). See also Gnedenko (1967), Çinlar (1972), and Cox and Isham (1980). This theorem explains the central role of Poisson processes in traffic flow theory. If each vehicle is described as a separate process, the arrival process at a given location is a superposition of thousands of component processes, which can be approximated as a Poisson process.

Another important result in the theory of point processes is that a superposition of two independent renewal processes is a renewal process if and only if

the components are Poisson processes (Çinlar 1972, Cox and Isham 1980). As we shall see below, this superimposed process is also a Poisson process. Memoryless component processes produce a memoryless superimposed process. If component processes are renewal processes, but their future depends on history (such as a minimum headway condition), all points in the superimposed process are no longer renewal points.

The number of points in a superimposed process during a (long) time interval is the sum of points in the component processes during the same time interval. Thus, the flow rate of a superimposed process is the sum of the component flow rates

$$q_{M,k} = \sum_{i \in I_k} q_i, \quad (3.29)$$

where I_k is the set of independent priority streams for minor stream k .

Let us assume a major road with multiple lanes ($i \in I_k$) of independent traffic streams which a minor stream k must give way to. When a minor stream vehicle reaches the first position in the queue at time τ , the lag is the time interval to the arrival of the first vehicle in any component process $i \in I_k$. The survival function of the lag distribution in the superimposed process is (Thompson 1988)

$$\begin{aligned} R_{Y,I_k}(v) &= \mathbb{P}\{Y_{I_k}(\tau) > v\} \\ &= \prod_{i \in I_k} \mathbb{P}\{Y_i(\tau) > v\} \\ &= \prod_{i \in I_k} R_{Y,i}(v), \end{aligned} \quad (3.30)$$

where $Y_i(\tau)$ is the lag in component process i starting from time τ , and $Y_{I_k}(\tau)$ is the lag in the superimposed process. The calculation of probability $\mathbb{P}\{Y_i(\tau) > v\}$ may require information about arrivals preceding time τ on lane (component process) i . If τ is the time of last arrival in the superimposed process, the headway is larger than t , if the headway on lane j (the lane of last arrival) and lags on other lanes are all larger than t ; i.e.,

$$\begin{aligned} R_{I_k,j}(t) &= \mathbb{P}\{T_j > t\} \prod_{\substack{i \in I_k \\ i \neq j}} \mathbb{P}\{Y_i(\tau) > t\} \\ &= R_j(t) \prod_{\substack{i \in I_k \\ i \neq j}} R_{Y,i}(t), \quad j \in I_k. \end{aligned} \quad (3.31)$$

The probability that a random point is from a component process j is the expected proportion of process j points in the superimposed process:

$$p_j = \frac{\lambda_j}{\sum_{i \in I_k} \lambda_i}. \quad (3.32)$$

The survival function of headways in the superimposed process can now be presented as

$$\begin{aligned}
 R_{I_k}(t) &= \sum_{j \in I_k} p_j \mathbb{P}\{T_j > t\} \prod_{\substack{i \in I_k \\ i \neq j}} \mathbb{P}\{Y_i(\tau) > t\} \\
 &= \sum_{j \in I_k} p_j R_j(t) \prod_{\substack{i \in I_k \\ i \neq j}} R_{Y,i}(t) \\
 &= \frac{1}{\sum_{i \in I_k} \lambda_i} \sum_{j \in I_k} \lambda_j R_j(t) \prod_{\substack{i \in I_k \\ i \neq j}} R_{Y,i}(t).
 \end{aligned} \tag{3.33}$$

If stream k has a stepwise gap acceptance function (3.6), the potential capacity is obtained following equation (3.8):

$$\begin{aligned}
 C_{p,k} &= 3600 \left(\sum_{i \in I_k} \lambda_i \right) \sum_{l=0}^{\infty} R_{I_k}(t_{c,k} + lt_{f,k}) \\
 &= \sum_{l=0}^{\infty} \sum_{j \in I_k} \left[q_j R_j(t_{c,k} + lt_{f,k}) \prod_{\substack{i \in I_k \\ i \neq j}} R_{Y,i}(t_{c,k} + lt_{f,k}) \right].
 \end{aligned} \tag{3.34}$$

It is assumed that there is no serial correlation between consecutive headways (see Luttinen 1996). Then the history of headways on lane j preceding time τ does not affect the probability $\mathbb{P}\{T_j > t\}$. It is also assumed that the history of other processes in I_k can be ignored, which is not always a realistic assumption.

In a Poisson process headways follow *negative exponential distribution*, and the Markov property simplifies equations considerably. Lag distributions on each lane are identical with headway distributions and they do not depend on previous history of the process:

$$\begin{aligned}
 \mathbb{P}\{Y_i(\tau) > v\} &= \mathbb{P}\{Y_i > v\} \\
 &= \mathbb{P}\{T_i > v\} \\
 &= e^{-\lambda_i v}.
 \end{aligned} \tag{3.35}$$

The survival function of headways in a superimposed process is

$$R_{I_k}(v) = \prod_{i \in I_k} e^{-\lambda_i v} = e^{-v \sum_{i \in I_k} \lambda_i}. \tag{3.36}$$

Because this is the negative exponential distribution with parameter $\lambda = \sum_{i \in I_k} \lambda_i$, the capacity of stream k can be calculated using equation (3.9) as follows:

$$C_{p,k} = \frac{e^{-t_c \sum_{i \in I_k} q_i / 3600} \sum_{i \in I_k} q_i}{1 - e^{-t_f \sum_{i \in I_k} q_i / 3600}}. \tag{3.37}$$

The theory of superimposed point processes gives credibility to Poisson process (exponential headways) as a gap availability model under multiple independent priority streams. The results also indicate that under Poisson assumption the superimposed process is mathematically no more complicated than the component processes. A superposition of Poisson processes is a Poisson process having as a parameter the sum of component process parameters.

If headways follow *shifted exponential distribution* the survival functions of headways and lags in the component processes are

$$R_i(t) = e^{-\theta_i(t-t_{p,i})}, \quad t_{p,i} < t \quad (3.38)$$

$$R_{Y,i}(t) = \frac{e^{-\theta_i(t-t_{p,i})}}{1 + \theta_i t_{p,i}}, \quad t_{p,i} < t. \quad (3.39)$$

The capacity of a minor stream yielding to several independent major streams having shifted exponential headway distributions can now be expressed as

$$\begin{aligned} C_{p,k} &= \sum_{l=0}^{\infty} \sum_{j \in I_k} \left[q_j R_j(t_{c,k} + lt_{f,k}) \prod_{\substack{i \in I_k \\ i \neq j}} R_{Y,i}(t_{c,k} + lt_{f,k}) \right] \\ &= \sum_{l=0}^{\infty} \sum_{j \in I_k} \left(q_j e^{-\theta_j(t_{c,k} + lt_{f,k} - t_{p,j})} \prod_{\substack{i \in I_k \\ i \neq j}} \frac{e^{-\theta_i(t_{c,k} + lt_{f,k} - t_{p,i})}}{1 + \theta_i t_{p,i}} \right) \\ &= \sum_{l=0}^{\infty} \frac{e^{-\sum_{i \in I_k} \theta_i(t_{c,k} + lt_{f,k} - t_{p,i})}}{\prod_{i \in I_k} (1 + \theta_i t_{p,i})} \sum_{j \in I_k} q_j (1 + \theta_j t_{p,j}) \\ &= \frac{e^{-\sum_{i \in I_k} \theta_i(t_{c,k} - t_{p,i})}}{\prod_{i \in I_k} (1 + \theta_i t_{p,i})} \left(\sum_{l=0}^{\infty} e^{-lt_{f,k} \sum_{i \in I_k} \theta_i} \right) \sum_{j \in I_k} q_j (1 + \theta_j t_{p,j}) \\ &= \frac{e^{-\sum_{i \in I_k} \theta_i(t_{c,k} - t_{p,i})} \sum_{i \in I_k} q_i (1 + \theta_i t_{p,i})}{(1 - e^{-t_{f,k} \sum_{i \in I_k} \theta_i}) \prod_{i \in I_k} (1 + \theta_i t_{p,i})}, \quad \forall i: t_{p,i} < t_{c,k}. \end{aligned} \quad (3.40)$$

Because the flow rate of a shifted exponential headway process can be expressed as

$$q_i = 3600 \lambda_i = \frac{3600 \theta_i}{1 + \theta_i t_{p,i}}, \quad (3.41)$$

the capacity of minor stream k is

$$\begin{aligned} C_{p,k} &= \frac{3600 e^{-\sum_{i \in I_k} \theta_i(t_{c,k} - t_{p,i})} \sum_{i \in I_k} \theta_i}{(1 - e^{-t_{f,k} \sum_{i \in I_k} \theta_i}) \prod_{i \in I_k} (1 + \theta_i t_{p,i})} \\ &= \frac{3600 \theta \prod_{i \in I_k} R_{Y,i}(t_{c,k})}{1 - e^{-\theta t_{f,k}}}, \quad \forall i: t_{p,i} < t_{c,k}, \end{aligned} \quad (3.42)$$

where $\theta = \sum_{i \in I_k} \theta_i$. If all minimum headways ($t_{p,i}$) are null, the result is equal to a capacity across random (Poisson) traffic streams.

Tanner (1967) and Troutbeck (1986) have derived the capacity of a minor stream crossing independent major streams each having a *Cowan's M3 headway distribution*. They assumed that minimum headways in all streams are identical. With the help of equation (3.34) the potential capacity can be obtained directly as:

$$\begin{aligned}
 C_{p,k} &= \sum_{l=0}^{\infty} \sum_{j \in I_k} \left(q_j R_j(t_{c,k} + lt_{f,k}) \prod_{\substack{i \in I_k \\ i \neq j}} R_{Y,i}(t_{c,k} + lt_{f,k}) \right) \\
 &= \sum_{l=0}^{\infty} \sum_{j \in I_k} \left(q_j \phi_j e^{-\gamma_j(t_{c,k} + lt_{f,k} - t_{p,j})} \prod_{\substack{i \in I_k \\ i \neq j}} \frac{\phi_i e^{-\gamma_i(t_{c,k} + lt_{f,k} - t_{p,i})}}{\phi_i + \gamma_i t_{p,i}} \right) \\
 &= \sum_{l=0}^{\infty} \frac{e^{-\sum_{i \in I_k} \gamma_i(t_{c,k} + lt_{f,k} - t_{p,i})}}{\prod_{i \in I_k} \left(1 + \frac{\gamma_i t_{p,i}}{\phi_i} \right)} \sum_{j \in I_k} q_j (\phi_j + \gamma_j t_{p,j}) \quad (3.43) \\
 &= \frac{e^{-\sum_{i \in I_k} \gamma_i(t_{c,k} - t_{p,i})}}{\prod_{i \in I_k} \left(1 + \frac{\gamma_i t_{p,i}}{\phi_i} \right)} \left(\sum_{l=0}^{\infty} e^{-lt_{f,k} \sum_{i \in I_k} \gamma_i} \right) \sum_{j \in I_k} 3600 \gamma_j \\
 &= 3600 \frac{e^{-\sum_{i \in I_k} \gamma_i(t_{c,k} - t_{p,i})} \sum_{i \in I_k} \gamma_i}{1 - e^{-t_{f,k} \sum_{i \in I_k} \gamma_i}} \prod_{i \in I_k} \frac{\phi_i}{\phi_i + \gamma_i t_{p,i}}, \quad \forall i: t_{p,i} < t_{c,k},
 \end{aligned}$$

where flow rate is expressed as

$$q_j = \frac{3600}{\mathbb{E}[T_j]} = \frac{3600 \gamma_j}{\phi_j + \gamma_j t_{p,j}}. \quad (3.44)$$

Equation (3.43) can also be expressed as

$$C_{p,k} = \frac{3600 \gamma \prod_{i \in I_k} R_{Y,i}(t_{c,k})}{1 - e^{-\gamma t_{f,k}}}, \quad (3.45)$$

where $\gamma = \sum_{i \in I_k} \gamma_i$. This is, in a more general form, the capacity equation of Tanner (1967). For a single priority stream the equation reduces to equation (3.13). Equation (3.43) can be used to obtain the capacity across multiple independent streams having any combination of negative exponential, shifted exponential, and M3 headway distributions.

The potential capacity across two streams i and j having Cowan's M3 headway distribution with equal minimum headways (t_p) is

$$C_{p,k} = \frac{3600 \phi_i \phi_j (\gamma_i + \gamma_j) e^{-(\gamma_i + \gamma_j)(t_{c,k} - t_p)}}{(\phi_i + \gamma_i t_p)(\phi_j + \gamma_j t_p) (1 - e^{-t_{f,k}(\gamma_i + \gamma_j)})}, \quad t_p < t_{c,k}. \quad (3.46)$$

This equation is identical with the equation 8 of Troutbeck (1986). Figure 3.11 displays a set of curves identical with Fig. 1 of Troutbeck (1986). It shows that major stream bunching increases minor stream capacity, even in the case of two independent priority streams.

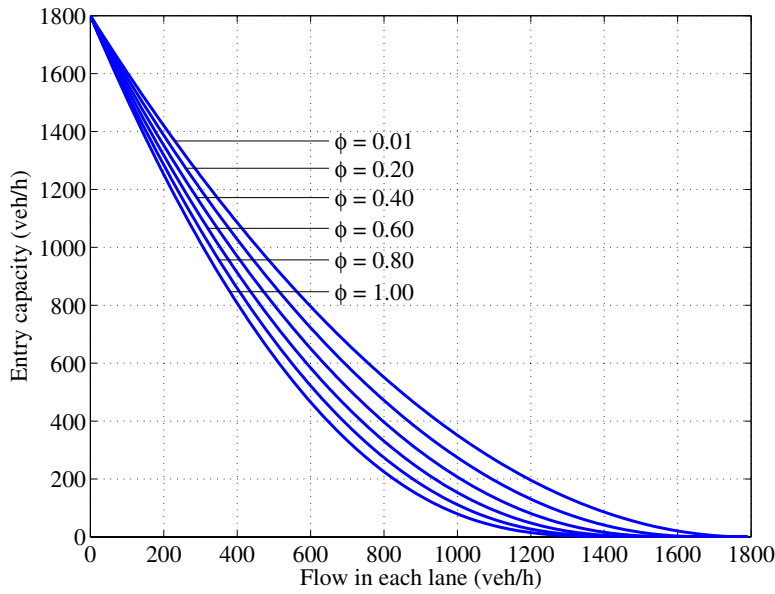


FIGURE 3.11 · Entry capacity at an intersection having two identical major streams with minimum headway $t_p = 2$ s and different proportions (ϕ) of free vehicles, and minor stream critical gap $t_c = 4$ s and follow-up time $t_f = 2$ s (see Troutbeck 1986)

3.2 Capacity of Rank 3 streams

3.2.1 Queuing model for Rank 3 stream capacity

Figure 3.12 displays a simplified presentation of traffic streams at a T-intersection. The minor road left turning vehicles (stream 7, Rank 3) must give way to streams 2 and 4. Streams 2 and 4 are, however, not independent. The Rank 2 stream 4 must yield to Rank 1 stream 2. Thus stream 7 vehicles can enter the intersection only if there is a gap larger than the critical gap in the combined major stream (2 and 4) *and* no stream 4 vehicles are waiting for entry. Models assuming independent priority streams (Section 3.1.2) would overestimate the capacity. A modified approach is required.

The time available for minor stream vehicles consists of two types of periods: *i*) major stream queuing and queue discharge periods, and *ii*) gap acceptance periods. During gap acceptance periods all priority stream vehicles can proceed without delay, and major streams can be considered independent. When the proportion of time available for minor stream gap acceptance, number of major stream headways, and the headway distribution in the superimposed process during the same time periods are estimated, the capacity of a minor stream can be calculated using the standard procedures described above. In order to do this, the queuing model must be evaluated in detail.

Figure 3.13 displays a queuing model description of Rank 2 stream i vehicles

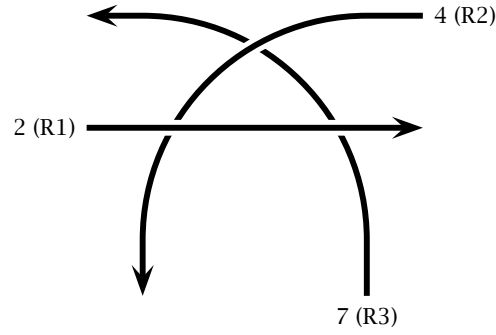


FIGURE 3.12 · Three ranks of streams at a T-intersection

blocking the entry of Rank 3 stream k vehicles. As vehicle 1 enters the queue of stream i , Rank 1 vehicles block the entry of both Rank 2 and Rank 3 vehicles. When the antiblock starts, vehicle 1 enters service, and the busy period of the server begins. During the service time $t_{f,i}$ no other stream i vehicles can enter the intersection (service). Vehicle 2 has to wait for the service of vehicle 1 to complete. As vehicle 1 departs, vehicle 2 enters service. Busy period ends as vehicle 2 departs service. The length of the busy period is $2t_{f,i}$. An isolated vehicle 3 arrives to an empty system. As it enters the queue, the service starts immediately. It blocks stream k for the time of the critical gap $t_{c,k}$.

The block of minor stream k terminates as the server of stream i becomes idle. After that the server of stream k starts service. The blocks and antiblocks in Figure 3.13 are described in terms of priority stream service departure times. Accordingly, the major stream arrival times in Figure 2.11 on page 35 should also be interpreted as service departure times. As Rank 1 flows decrease, the Rank 2 stream approaches an M/D/1 queuing system, and the departure flow approaches the model of Tanner (see page 33).

The queuing analogy should not be stretched too far. When an acceptable gap follows a stream i vehicle, a stream k vehicle does not—safety permitting—have to wait for $t_{f,i}$ before it can enter the intersection. The possibility of overlapping service times between conflicting streams cannot be excluded.

In the basic queuing model of Figure 3.13 each stream has a separate server with service time equal to the follow-up time. Service is interrupted by blocks created by higher priority vehicles. If interruptions in the operation of stream i server are included in the service time, the average service time is the inverse of capacity (C_i^{-1}). As Figure 3.14 displays, in this model vehicle 1 enters service immediately, and service time includes the time spent waiting for the block termination. Service begins when a vehicle enters the stop line and terminates $t_{f,i}$ after the vehicle has crossed the stop line. Vehicle 1 departs service $t_{f,i}$ after the termination of the block, and the next vehicle enters service. Major stream arrivals frequently extend the service times of minor stream vehicles beyond the follow-up time.

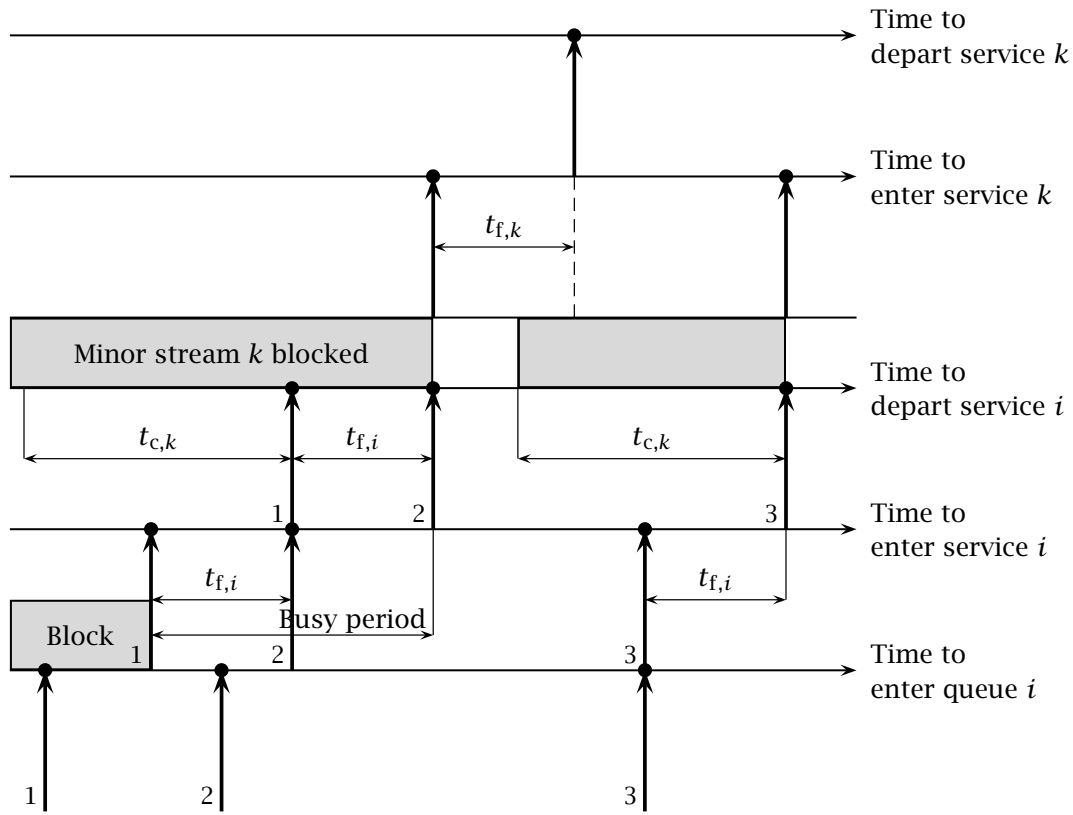


FIGURE 3.13 · A queuing model having service interrupted by blocks

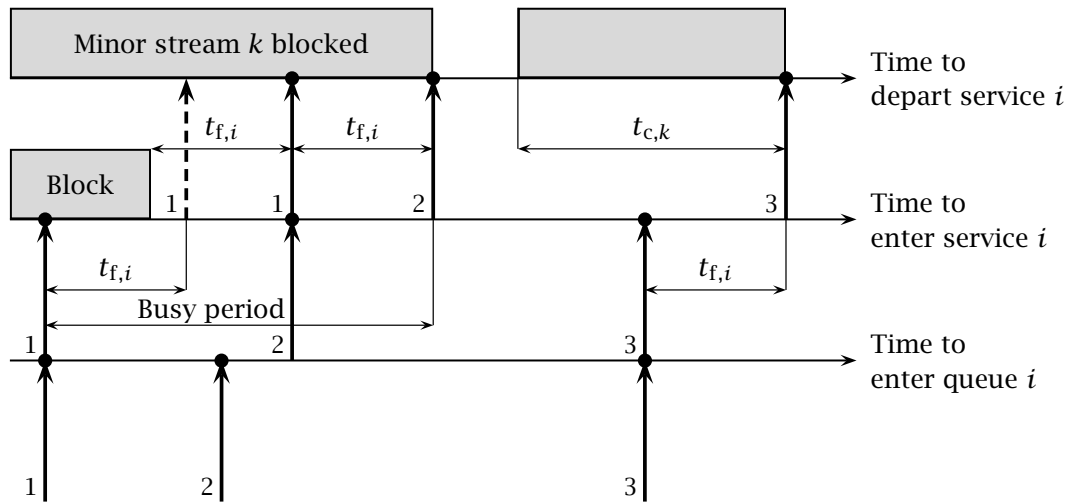


FIGURE 3.14 · A queuing model having service times extended by blocks

In both models the service departure times of stream i vehicles are equal. In both models the busy period ends as server i becomes idle. During busy periods the entry of stream i vehicles may be blocked by Rank 1 vehicles. The starting time of a busy period is, however, different in the models. The model in Figure 3.14 includes the time that the first vehicle is waiting for an antiblock in the busy period, while this time is excluded from the busy period in model 3.13. Because a Rank 2 vehicle waiting for entry prevents the entry of a Rank 3 vehicle, this time period is not available for gap acceptance. Accordingly, the capacity of a Rank 3 stream should be estimated using the model in Figure 3.14. This is the model used in the discussion below.

The estimation of Rank 3 stream k capacity proceeds in stages:

1. The potential capacity ($C_{p,i}$) of Rank 2 stream i is determined using the procedures described earlier.
2. An adjustment factor ($f_{a,i}$, fraction of time available for gap acceptance) is evaluated using $C_{p,i}^{-1}$ as the average service time of stream i server.
3. The gap acceptance capacity $C_{a,k}$ of stream k is estimated assuming that no stream i vehicles are waiting for entry and all major streams are independent.
4. The movement capacity of stream k is the gap acceptance capacity multiplied by the adjustment factor

$$C_{m,k} = f_{a,i} C_{a,k}, \quad (3.47)$$

or

$$(3.48)$$

$$C_{m,k} = f_i C_{p,k}, \quad (3.49)$$

where f_i is the impedance factor describing the fraction of potential capacity available as movement capacity.

The estimation of factors $f_{a,i}$ and f_i , and gap-acceptance capacity $C_{a,k}$ is discussed next, but first the Rank 2 queue discharge is described in macroscopic terms.

3.2.2 Rank 2 queue discharge

The utilization factor of a server in a Rank 2 stream i is

$$\rho_i = \frac{q_i}{C_{p,i}}, \quad (3.50)$$

where the potential capacity $C_{p,i}$ is calculated using equation (3.9) above. Utilization factor ρ_i is the fraction of time that server i is busy under the potential capacity conditions. The fraction of randomly arriving (Poisson) vehicles in stream

i having to wait for service is equal to the fraction of vehicles arriving when the server is busy, which is equal to the utilization factor ρ_i (Khintchine 1960). These vehicles discharge from a queue and their headways cannot be used by minor stream drivers. Other $(1 - \rho_i)$ vehicles enter the intersection without queuing, although they may have had to wait for an antiblock at the stop line.

The number of queue discharge headways per hour is $\rho_i q_i$. Because the average queue discharge headway is $C_{p,i}^{-1}$, the fraction of time required for queue discharge is $\rho_i q_i C_{p,i}^{-1} = \rho_i^2$. During queue discharge the departure flow rate of stream i is $C_{p,i} = q_i \rho_i^{-1}$. The flow rate can be expressed as the sum of flows during the queue discharge and free departure periods:

$$q_i = \rho_i^2 \frac{q_i}{\rho_i} + (1 - \rho_i^2) q_{a,i}, \quad (3.51)$$

where $q_{a,i}$ is the average flow rate during the free departure periods. The vehicles departing during the free departure periods did not have to wait for service, although the service times may have been extended by higher priority vehicles. The average flow rate during the free departure periods can be solved from equation (3.51) as

$$q_{a,i} = \frac{q_i}{1 + \rho_i}. \quad (3.52)$$

The average free departure headway of stream i can now be expressed as

$$\bar{t}_{a,i} = \frac{3600}{q_{a,i}} = 3600 \left(\frac{1}{q_i} + \frac{1}{C_{p,i}} \right). \quad (3.53)$$

It is the sum of average arrival headway and average service time. If Rank 1 flows approach null, $C_{p,i}^{-1}$ approaches $t_{f,i}$ and the average free departure headway in stream i approaches $\lambda_i^{-1} + t_{f,i}$.

Following Figure 3.15 an observation period τ of stream i can be divided into two distinct time periods: queue discharge and free departures. During a queue-discharge period of length $\rho_i^2 \tau$ the departure flow rate is $C_{p,i}$. The free departure period lasts for $(1 - \rho_i^2) \tau$, and has flow rate $q_i (1 + \rho_i)^{-1}$.

As presented in equation (3.4), capacity is the product of expected departures during a randomly selected headway multiplied by the expected number of headways per time unit. The number of queue discharge headways in stream i is $\rho_i q_i$. The number of headways available for gap acceptance is $q_M = (1 - \rho_i) q_i$. Because these vehicles have found the server idle, no headway can be shorter than the follow-up time (Dunne and Buckley 1972, Siegloch 1973). The headway distribution should be modified as $f_i(t \mid T \geq t_{f,i})$.

3.2.3 Rank 3 capacity with no Rank 1 conflict

Dunne and Buckley (1972) have studied the capacity of a Rank 3 stream with no Rank 1 conflict, but Rank 2 stream modified by filtering through a Rank 1 stream. A similar model is discussed here, but in this model a Rank 3 vehicle cannot proceed, if a Rank 2 vehicle is waiting for entry.

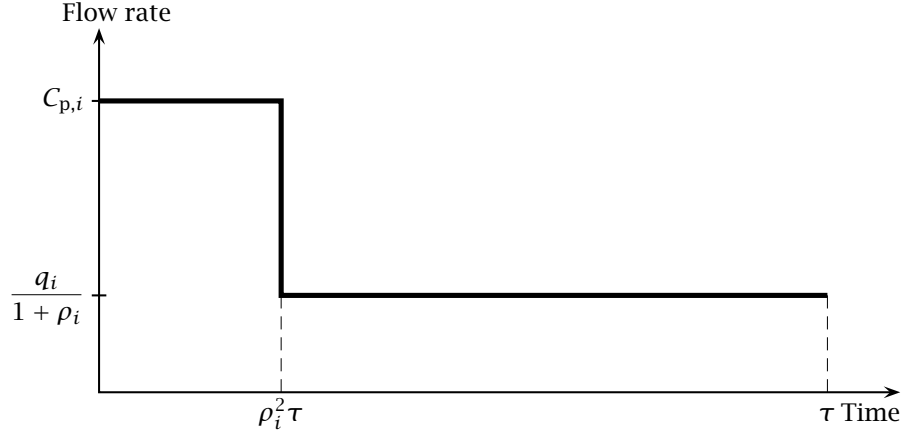


FIGURE 3.15 · Flow rates during queue-discharge and free-flow periods

If Rank 3 vehicles can ignore other priority streams besides stream i (as in Fig. 3.16), the movement capacity of a Rank 3 stream k is:

$$\begin{aligned}
 C_{m,k} &= (1 - \rho_i) q_i \int_0^\infty \alpha(t) f_i(t \mid T \geq t_{f,i}) dt \\
 &= \frac{(1 - \rho_i) q_i}{R_i(t_{f,i})} \int_0^\infty \alpha(t) f_i(t) dt \\
 &= (1 - \rho_i) \frac{C_{p,k}}{R_i(t_{f,i})},
 \end{aligned} \tag{3.54}$$

where $C_{p,k}$ is the potential capacity obtained by the methods above. For a stepwise gap acceptance function (3.6) the capacity is obtained as

$$\begin{aligned}
 C_{m,k} &= (1 - \rho_i) q_i \sum_{j=0}^\infty R_i(t_{c,k} + j t_{f,k} \mid T \geq t_{f,i}) \\
 &= \frac{(1 - \rho_i) q_i}{R_i(t_{f,i})} \sum_{j=0}^\infty R_i(t_{c,k} + j t_{f,k}).
 \end{aligned} \tag{3.55}$$

Example 3.1 (No Rank 1 flows) For random (exponential) headways in stream i and no Rank 1 flow the movement capacity of stream k is

$$\begin{aligned}
 C_{m,k} &= \frac{(1 - \rho_i) q_i}{e^{-\lambda_i t_{f,i}}} \times \frac{e^{-\lambda_i t_{c,k}}}{1 - e^{-\lambda_i t_{f,k}}} \\
 &= \frac{(1 - \rho_i) q_i e^{-\lambda_i (t_{c,k} - t_{f,i})}}{1 - e^{-\lambda_i t_{f,k}}}.
 \end{aligned} \tag{3.56}$$

The equation has the same form as Tanner's capacity formula (3.18). This was to be expected, because in the absence of Rank 1 flows $\rho_i = \lambda_i t_{f,i}$, and the Rank 2 flow i is a departure process from an M/D/1 queuing system (see also Dunne and Buckley 1972). Constrained headways have a fixed length, and unconstrained headways follow the shifted exponential distribution. ■

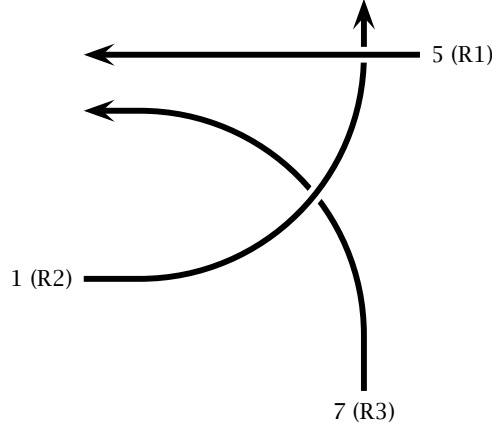


FIGURE 3.16 · Three ranks of streams with no conflict between Ranks 1 and 3

3.2.4 Time available for gap acceptance

Harders (1968) proposed that when no Rank 2 vehicles are queuing, the movement capacity of Rank 3 streams can be estimated assuming independent priority streams.³ When there is a queue of Rank 2 stream i vehicles, no Rank 3 stream k vehicles can enter the intersection. Thus the movement capacity of stream k can be expressed as

$$C_{m,k} = p_{0,i} C_{p,k}, \quad (3.57)$$

where $p_{0,i}$ is the probability of empty queue in stream i , and $C_{p,k}$ is the potential capacity of stream k assuming independent priority streams.

Harders (1968) observed that flow rate in a Rank 2 stream i can be expressed as the sum of delayed and undelayed vehicles. During a proportion of time $1 - p_{0,i}$ vehicles discharge from queue at flow rate $C_{p,i}$. When there is no queuing, a vehicle can enter the intersection without delay, if there is an antiblock in the priority stream j and the headway in stream i is longer than the follow-up time $t_{f,i}$. Assuming Poisson arrivals Harders expressed flow rate q_i as

$$q_i = (1 - p_{0,i}) C_{p,i} + p_{0,i} \chi q_i, \quad (3.58)$$

where χ is the probability of an antiblock in priority stream j and a headway larger than the follow-up time in stream i :

$$\chi = e^{-t_{c,i} q_j / 3600} e^{-t_{f,i} q_i / 3600} = e^{(t_{c,i} q_j + t_{f,i} q_i) / 3600}. \quad (3.59)$$

The probability of empty queue in stream i was then obtained as

$$p_{0,i} = \frac{C_{p,i} - q_i}{C_{p,i} - \chi q_i}. \quad (3.60)$$

³This assumption is correct for Poisson arrivals only. As Rank 2 queue clears, the first vehicle in a Rank 3 stream queue faces not a gap but a lag.

Harders called $p_{0,i}$ an *impedance factor*. Figure 10-5 in HCM 1985 (Transportation Research Board 1985) is based on this equation (see Brilon 1988a).

In HCM2000 (Transportation Research Board 2000) the impedance factor $p_{0,i}$ is the probability that at a random time instant the system in stream i is empty; i.e.,

$$p_{0,i} = 1 - \rho_i = 1 - \frac{q_i}{C_{p,i}}, \quad \rho_i \leq 1, \quad (3.61)$$

where ρ_i is the utilization factor (degree of saturation) of the queuing system in stream i , q_i is the arrival flow rate, and $C_{p,i}$ is the potential capacity of stream i . Brilon (1988a) and Grossmann (1991) have demonstrated that equation (3.61) gives acceptable results. Equation (3.57) is used to calculate the movement capacity.

This method has two problems:

1. The impedance factor $p_{0,i}$ is interpreted as the fraction of time available for gap acceptance, whereas it should be interpreted as the fraction of headways available for gap acceptance.
2. The distribution of Rank 2 stream headways during the gap acceptance periods should be modified to exclude headways shorter than the follow-up time $t_{f,i}$ and to adjust the flow rate during free-departure conditions (Siegloch 1973).

These modifications are discussed below.

In queuing models 3.13 and 3.14 Rank 2 vehicles enter the intersection as they depart the service. The critical gaps for Rank 3 vehicles are evaluated backward from these time instants. The critical gap of a minor stream includes the “service times” of priority stream vehicles. Thus, the blocking effect of vehicle 1 in Figure 3.14 does not start as a vehicle enters service, but from the “virtual departure time” (dashed arrow) that it would have departed service if the entry was not blocked by Rank 1 vehicles. It is also obvious, that the blocking effect of an isolated vehicle 3 is $t_{c,k}$, not $t_{c,k} + t_{f,i}$. The follow-up time of an isolated vehicle should be included as time available for Rank 3 gap acceptance. The time period blocked by a freely departing vehicle is equal to the critical gap, and it is considered in the gap acceptance process.⁴ This is evident, if we consider a situation having no Rank 1 flow and no queuing in a Rank 2 stream. The impedance factor should be equal to unity. Consequently, the *time period blocked by queuing Rank 2 vehicles is the busy period minus the follow-up time*.

If a busy period in stream i ends at time τ the following idle time is the lag $Y_i(\tau)$ remaining at time τ . For random (Poisson) arrivals the average idle period is equal to the average lag, which is equal to the average headway λ_i^{-1} . If the average length of a busy period is $\mathbb{E}[T_{bp,i}]$ and the average length of an idle period is $\mathbb{E}[T_{ip,i}] = \lambda_i^{-1}$, then for a queuing process with Poisson arrivals holds

⁴Akçelik (1994) includes the follow-up times of all major stream vehicles in antiblocks, but subtracts follow-up time from critical gap.

(Gross and Harris 1974)

$$\frac{\rho_i}{1 - \rho_i} = \frac{\mathbb{E}[T_{bp,i}]}{\mathbb{E}[T_{ip,i}]} = \lambda_i \mathbb{E}[T_{bp,i}]. \quad (3.62)$$

The expected length of a busy period is now obtained as

$$\mathbb{E}[T_{bp,i}] = \frac{\rho_i}{\lambda_i(1 - \rho_i)}. \quad (3.63)$$

The average busy cycle is the expected sum of busy and idle periods:

$$\mathbb{E}[T_{bc,i}] = \mathbb{E}[T_{bp,i}] + \mathbb{E}[T_{ip,i}] = \frac{\rho_i}{\lambda_i(1 - \rho_i)} + \frac{1}{\lambda_i} = \frac{1}{\lambda_i(1 - \rho_i)}. \quad (3.64)$$

The proportion of randomly arriving (Poisson) vehicles having to wait for service is the proportion of vehicles arriving during a busy period (see Khintchine 1960):

$$p_{w,i} = \frac{\lambda_i \mathbb{E}[T_{bp,i}]}{\lambda_i \mathbb{E}[T_{bc,i}]} = \rho_i. \quad (3.65)$$

This proportion does not include the vehicles, whose service initiated the busy periods.

The proportion of time available in a Rank 1 stream for Rank 3 gap acceptance is obtained as

$$\begin{aligned} f_{a,i} &= 1 - \frac{\mathbb{E}[T_{bp,i}] - t_{f,i}}{\mathbb{E}[T_{bc,i}]} \\ &= 1 - \rho_i + \frac{t_{f,i}}{\mathbb{E}[T_{bc,i}]} \\ &= (1 + \lambda_i t_{f,i})(1 - \rho_i) \\ &= \left(1 + \frac{q_i t_{f,i}}{3600}\right) \left(1 - \frac{q_i}{C_i}\right). \end{aligned} \quad (3.66)$$

This is the proportion of time during which queuing stream i vehicles do not block the entry of Rank 3 vehicles; i.e., the proportion of time during which the Rank 2 departure headways (real and virtual) follow the shifted exponential distribution. As Rank 2 capacity approaches $t_{f,i}^{-1}$ the impedance factor approaches the probability $1 - \rho_i^2$ of no queue (at most one customer in system) in an M/G/1 queuing system.⁵ If stream i is congested ($\rho_i > 1$), the server does not have idle periods and the adjustment factor is null. In order to avoid negative values of $f_{a,i}$ the equation should be slightly modified:

$$f_{a,i} = \left(1 + \frac{q_i t_{f,i}}{3600}\right) \left(1 - \frac{q_i}{C_i}\right)_+. \quad (3.67)$$

⁵Kendall (1951) has shown that for Poisson arrivals the probability of a queue length left behind by a departing customer is geometric: $p_n = (1 - \rho)\rho^n$. Thus the probability that a departing customer leaves behind at most one customer is $(1 - \rho) + (1 - \rho)\rho = 1 - \rho^2$. For Poisson arrivals this is also the queue size distribution for a outside observer (see Cooper 1981); i.e. the probability that at a random time instant no vehicles are waiting for service.

As Figure 3.17 displays, the factor $f_{a,i}$ assigns a larger proportion of time to gap acceptance than $p_{0,i} = 1 - \rho_i$. The difference between $p_{0,i}$ and $f_{a,i}$ increases as potential capacity $C_{p,i}$ increases, because then busy cycles become shorter.

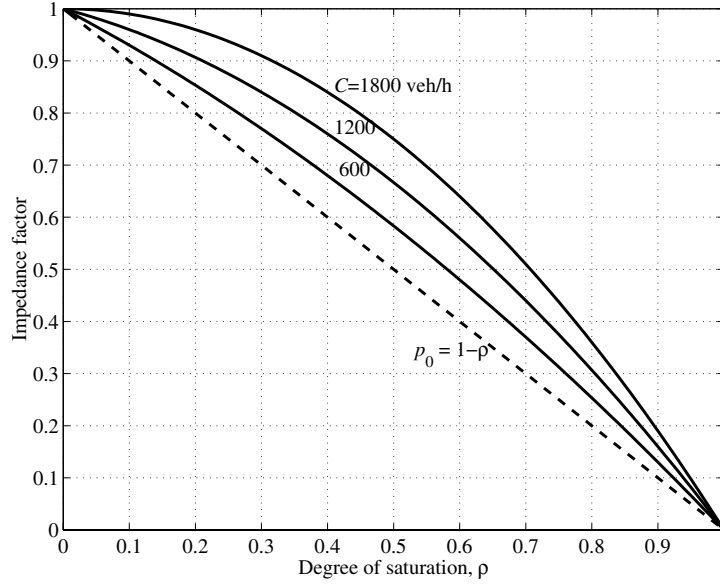


FIGURE 3.17 · Two impedance factors: p_0 (dashed line) and f_a (solid curves)

In Figure 3.17 the follow-up time is $t_{f,i} = 2$ seconds. Capacity 1,800 veh/h is attained, when there is no conflicting higher priority traffic. Even then the impedance factor decreases to zero as the degree of saturation increases to unity. This indicates that the model accounts for a bottleneck effect. Because the discharge headway of vehicles entering the intersection cannot be shorter than $t_{f,i}$, a turning movement may have queue formation even though there is no conflicting higher priority traffic. Furthermore, at low degrees of saturation the drop in capacity is not as steep as $p_{0,i}$ indicates.

Because $f_{a,i}$ is the time available for gap acceptance, the number of Rank 1 stream j headways available for gap acceptance is $f_{a,i}q_j$. As noted above, the number of Rank 2 stream i headways available is $(1 - \rho_i)q_i$. The adjustments are different, because the time periods available for Rank 3 gap acceptance are random for the Rank 1 stream but not for the Rank 2 stream.

3.2.5 Gap availability for Rank 3 vehicles

The discussion above has demonstrated that the fraction of Rank 2 stream i headways available for Rank 3 gap acceptance is $1 - \rho_i$. These may be headways of either real or virtual stream i departures. The fraction of time $f_{a,i}$ available for gap acceptance is given by equation (3.67), which also gives the fraction of Rank

1 stream j headways available for gap acceptance. The total number of available headways per hour is

$$q_{a,M} = (1 - \rho_i)q_i + f_{a,i}q_j = (1 - \rho_i) \left[q_i + \left(1 + \frac{q_i t_{f,i}}{3600} \right) q_j \right]. \quad (3.68)$$

In order to derive the equation for movement capacity, it is necessary to obtain the distribution of headways during the gap acceptance periods. As indicated earlier these are headways of either real or virtual priority stream departures.

Traffic flow is described as a point process, where the location of a point on a time axis describes the departure time of a vehicle from a service process. A Rank 3 vehicle observes two point processes: process 'I' for Rank 1 departures and process 'II' for Rank 2 departures. Let us assume that all arrivals can be described as a Poisson process. Process I is independent of other streams, so that its headways follow the negative exponential distribution with location parameter λ_I . The headways available for gap acceptance in process II follow shifted exponential distribution with scale parameter λ_{II} , which is equal to the scale parameter of the exponential arrival headway distribution (see eq. 2.19). The location parameter (minimum headway) is equal to the follow-up headway $t_p = t_{f,II}$. Because departure times of delayed vehicles have been replaced by virtual departure times, the virtual flow rate of stream II during gap acceptance periods is

$$q_{a,II} = 3600\lambda_{a,II} = \frac{3600\lambda_{II}}{1 + \lambda_{II}t_{f,2}}, \quad (3.69)$$

which is the flow rate of a shifted exponential headway process.

Because all service times in the Rank 2 stream exceeding $t_{f,2}$ have been excluded, these two processes can be considered independent. The process observed by the Rank 3 vehicles is the superposition of processes 1 and 2. The survival function of intervals (headways) in a superimposed process of two component processes is

$$R(t) = p_I R_I(t) R_{Y,II}(t) + p_{II} R_{II}(t) R_{Y,I}(t), \quad (3.70)$$

The proportions of points from both component processes are

$$p_I = \frac{\lambda_I}{\lambda_I + \lambda_{a,II}} = \frac{\lambda_I(1 + \lambda_{II}t_p)}{\lambda_I(1 + \lambda_{II}t_p) + \lambda_{II}} \quad (3.71)$$

$$p_{II} = \frac{\lambda_{a,II}}{\lambda_I + \lambda_{a,II}} = \frac{\lambda_{II}}{\lambda_I(1 + \lambda_{II}t_p) + \lambda_{II}}. \quad (3.72)$$

where $\lambda_{a,II} = \lambda_{II}(1 + \lambda_{II}t_p)^{-1}$. The same results are obtained, if p_I and p_{II} are calculated with $\lambda_{a,I} = f_{a,II}\lambda_I = (1 + \lambda_{II}t_p)(1 - \rho_{II})\lambda_I$ and $\lambda_{a,II} = (1 - \rho_{II})\lambda_{II}$. The survival function can now be expressed as

$$\begin{aligned} R(t) &= \left(\frac{p_I}{1 + \lambda_{II}t_p} + p_{II} \right) e^{-(\lambda_I + \lambda_{II})t + \lambda_{II}t_p} \\ &= \frac{\lambda_I + \lambda_{II}}{\lambda_I(1 + \lambda_{II}t_p) + \lambda_{II}} e^{-(\lambda_I + \lambda_{II})t + \lambda_{II}t_p}, \quad t \geq t_p. \end{aligned} \quad (3.73)$$

If $t_p = 0$, we get the survival function of negative exponential distribution with scale parameter $\lambda = \lambda_I + \lambda_{II}$, as expected.

3.2.6 Movement capacity under one Rank 2 stream

When minor stream gap acceptance follows the step function (3.6), the movement capacity of a Rank 3 stream k can be obtained as

$$\begin{aligned}
 C_{m,k} &= q_M \sum_{i=0}^{\infty} R(t_{c,k} + it_{f,k}) \\
 &= \frac{3600(1 - \rho_{II})(\lambda_I(1 + \lambda_{II}t_p) + \lambda_{II})(\lambda_I + \lambda_{II})}{\lambda_I(1 + \lambda_{II}t_p) + \lambda_{II}} \sum_{i=0}^{\infty} e^{-(\lambda_I + \lambda_{II})(t_{c,k} + it_{f,k}) + \lambda_{II}t_p} \\
 &= 3600(1 - \rho_{II})(\lambda_I + \lambda_{II})e^{-(\lambda_I + \lambda_{II})t_{c,k} + \lambda_{II}t_p} \sum_{i=0}^{\infty} e^{-(\lambda_I + \lambda_{II})it_{f,k}} \\
 &= 3600(1 - \rho_{II})(\lambda_I + \lambda_{II}) \frac{e^{-(\lambda_I + \lambda_{II})t_{c,k} + \lambda_{II}t_p}}{1 - e^{-(\lambda_I + \lambda_{II})t_{f,k}}} \\
 &= \boxed{(1 - \rho_{II})(q_I + q_{II}) \frac{e^{[-(q_I + q_{II})t_{c,k} + q_{II}t_p]/3600}}{1 - e^{-(q_I + q_{II})t_{f,k}/3600}}}, \quad t_p < t_{c,k} \\
 &= f_{II}C_{p,k},
 \end{aligned} \tag{3.74}$$

where

$$\boxed{f_{II} = \frac{(1 - \rho_{II})}{e^{-q_{II}t_p/3600}}} \tag{3.75}$$

is the impedance factor due to Rank 2 stream II queuing, $t_p = t_{f,II}$ is the follow-up time of the Rank 2 stream, and $C_{p,k}$ is the potential capacity (3.9) assuming a Poisson priority stream with flow rate $q_I + q_{II}$, critical gap $t_{c,k}$, and follow-up time $t_{f,k}$. The Rank 1 priority streams used in the analysis of ρ_{II} may be different than the Rank 1 priority streams of stream k .

The numerator in the impedance factor is the probability that the Rank 2 stream has no waiting or discharging vehicles. The denominator is the probability of a headway or lag larger than t_p in the Poisson process of the Rank 2 stream. It adjusts the capacity for the filtered headway distribution (and lower flow rate), of the Rank 2 stream during the periods available for Rank 3 gap acceptance. The result is similar to equation (3.55). The relation between f_{II} and the HCM2000 impedance factor $p_{0,II} = (1 - \rho_{II})_+$ is

$$f_{II} = \frac{p_{0,II}}{e^{-q_{II}t_{f,II}/3600}}. \tag{3.76}$$

If $q_{II} = 0$, equation (3.74) is equal to the capacity equation (3.9) for Poisson priority flow. If $q_I = 0$, equation (3.74) is equal to Tanner's formula (3.18), as indicated in example 3.1 on page 64.

Capacity equation (3.74) can be obtained directly from equation (3.40) by setting number of headways $q_{a,I} = (1 + \lambda_{II}t_p)(1 - \rho_{II})q_I$, $q_{a,II} = (1 - \rho_{II})q_{II}$, scale

parameters $\theta_I = \lambda_I$, $\theta_{II} = \lambda_{II}$, and minimum headways $t_{f,I} = 0$ and $t_{f,II} = t_p$:

$$C_{m,k} = \frac{e^{-(\lambda_I + \lambda_{II})t_{c,k} + \lambda_{II}t_p} (q_I + q_{II}) (1 + \lambda_{II}t_p) (1 - \rho_{II})}{(1 - e^{-(\lambda_I + \lambda_{II})t_{f,k}}) (1 + \lambda_{II}t_p)} \quad (3.77)$$

$$= (1 - \rho_{II})(q_I + q_{II}) \frac{e^{[-(q_I + q_{II})t_{c,k} + q_{II}t_p]/3600}}{1 - e^{-(q_I + q_{II})t_{f,k}/3600}}, \quad t_p < t_{c,k}$$

Figure 3.18 displays the capacity of a Rank 3 stream with different Rank 2 and total $(q_I + q_{II})$ flow rates. The critical gap of both Rank 2 and Rank 3 vehicles is five seconds and follow-up time three seconds. The capacities of equation (3.74) are compared with results of Monte Carlo simulations.

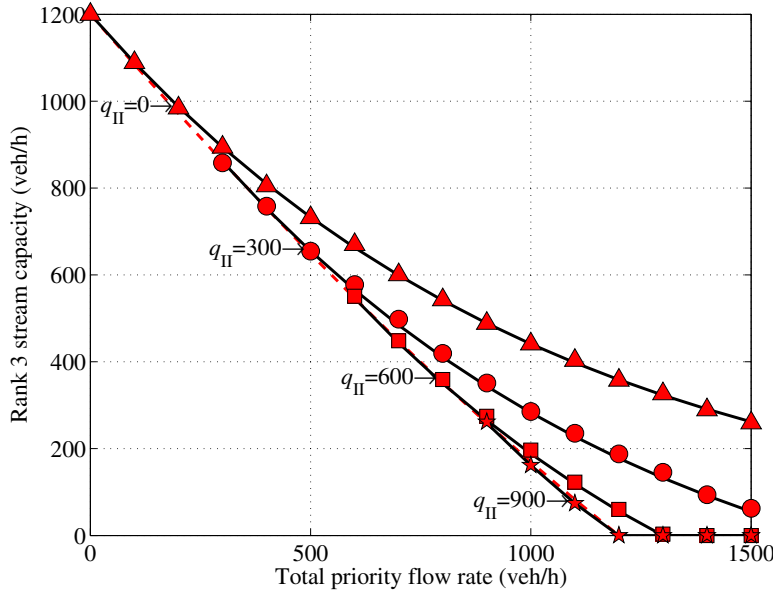


FIGURE 3.18 • Movement capacity of a Rank 3 stream with different Rank 2 (q_{II}) and total priority flow rates. Critical gaps and follow-up times of both Rank 2 and Rank 3 streams are 5 s and 3 s, respectively. Capacities were calculated with equation 3.74 (curves) and Monte Carlo simulation (markers). Dashed curve shows Tanner's capacity (3.18) with no Rank 1 flow and Rank 2 stream minimum headway equal to the follow-up time.

If there are no priority flows, the capacity is $3600/t_{f,k} = 1200$ veh/h. For zero Rank 2 flows Rank 3 capacity is obtained with the equation (3.9) for Poisson priority flow, displayed as the upper curve in Figure 3.18. For zero Rank 1 flows the capacity is obtained with Tanner's capacity formula (3.18), displayed as a dashed curve in Figure 3.18. When $q_I = 0$ veh/h and $q_{II} = 1200$ veh/h, no headways are available for Rank 3 gap acceptance, and Rank 3 capacity is null. The Rank 3 capacities for other proportions of Rank 1 and Rank 2 flows are found approximately between the capacity formulas (3.9) and (3.18).

Figure 3.19 displays a comparison of equation (3.74) with HCM2000 capacities. Dashed curves display the respective capacities with no Rank 1 flows. This is the major difference between equation (3.74) and HCM2000. The latter excludes the follow-up times of all Rank 2 vehicles from the time available for Rank 3 gap acceptance and assumes exponential priority stream headways. Equation (3.74) includes the follow-up times of non-queuing Rank 2 vehicles in the time available for Rank 3 gap acceptance and assumes shifted exponential priority stream headways that give lower Rank 2 flow rate during the gap acceptance periods than the average arrival rate of the (exponential) Rank 2 stream. Consequently, HCM2000 gives much lower capacities at low Rank 1 flows.

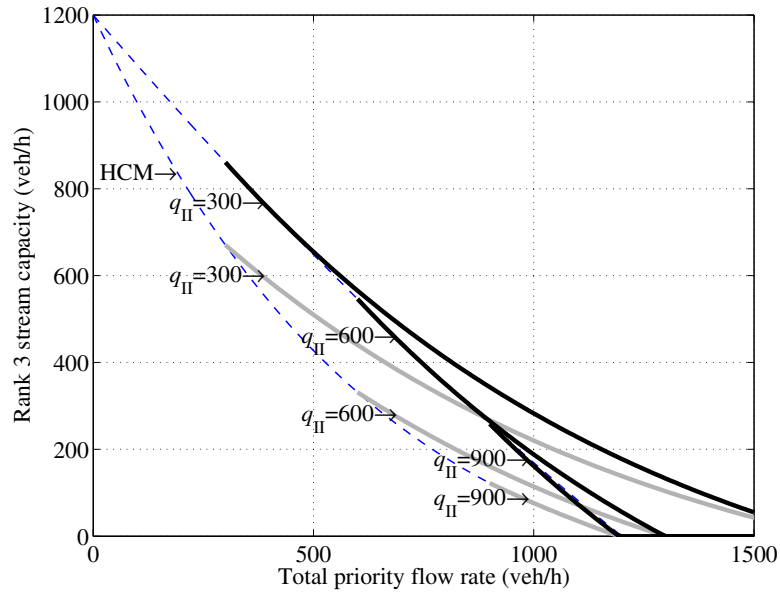


FIGURE 3.19 · Movement capacity of a Rank 3 stream: Equation 3.74 (black curves) compared with HCM2000 capacities (gray curves). Dashed curves display capacities with no Rank 1 flows.

Stream 7 at a T-intersection (Fig. 2.3) must give way to Rank 1 streams 2 and 5, and Rank 2 stream 4. Movement capacity is calculated as

$$C_{m,7} = f_4 C_{p,7}, \quad (3.78)$$

where

$$f_4 = \frac{1 - \frac{q_4}{C_{p,4}}}{e^{-q_4 t_{f,4}/3600}}. \quad (3.79)$$

The priority streams of stream 4 used in the calculation of $C_{p,4}$ are $I_4 = \{2, 3\}$. Potential capacity $C_{p,7}$ is calculated assuming priority streams $I_7 = \{2, 4, 5\}$. Rank 1 priority flow is $q_2 + q_5$ plus the possible effect of the major road right-turn stream 3. Rank 2 stream is q_4 .

3.2.7 Movement capacity under two Rank 2 streams

In a four-leg intersection (Fig. 2.2, page 19) minor road through streams 8 and 11 have Rank 3. They must give way to Rank 2 streams 1 and 4 (major road left turns). Stream 1 must give way to Rank 1 streams 5 and 6, and stream 4 must give way to streams 5 and 6. The priority flows of streams 8 and 11 during the time available for gap acceptance are $I_8 = \{1, 2, 4, 5, 6\}$ and $I_{11} = \{1, 2, 3, 4, 5\}$.

The impedance factor f_4 of major road left-turn stream 4 is given above. The impedance factor of stream 1 is likewise

$$f_1 = \frac{1 - \rho_1}{e^{-q_1 t_{f,1}/3600}} = \frac{1 - \frac{q_1}{C_{p,1}}}{e^{-q_1 t_{f,1}/3600}}. \quad (3.80)$$

It is the proportion of capacity available due to major stream queuing. Because the queuing processes of streams 1 and 4 are independent, it can be assumed that the combined impedance factor is the product

$$f_{1 \wedge 4} = f_1 f_4. \quad (3.81)$$

This is indeed the case, as will be shown next.

If the other Rank 2 stream is ignored, the proportion of time available for gap acceptance in streams 1 and 4 is

$$\begin{aligned} f_{a,1} &= \left(1 + \frac{q_1 t_{f,1}}{3600}\right) \left(1 - \frac{q_1}{C_1}\right), & q_1 < C_1 \\ f_{a,4} &= \left(1 + \frac{q_4 t_{f,4}}{3600}\right) \left(1 - \frac{q_4}{C_4}\right), & q_4 < C_4. \end{aligned} \quad (3.82)$$

These factors are probabilities that at a random time instant gap acceptance is not restricted by queuing vehicles in the respective stream. Because the queuing processes are independent, the probability that at a random time instant gap acceptance is not restricted by queuing vehicles in either stream is the product

$$f_{a,1 \wedge 4} = f_{a,1} f_{a,4}. \quad (3.83)$$

Because this is also the proportion of vehicles in Rank 1 streams arriving during gap acceptance periods, the number of vehicles observed in these streams during the gap acceptance periods is

$$\begin{aligned} q_{a,2} &= f_{a,1 \wedge 4} q_2 \\ q_{a,3} &= f_{a,1 \wedge 4} q_3 \\ q_{a,5} &= f_{a,1 \wedge 4} q_5 \\ q_{a,6} &= f_{a,1 \wedge 4} q_6. \end{aligned} \quad (3.84)$$

Of the “free headways” in stream 1 (4) only headways of vehicles arriving during the non-queuing periods of stream 4 (1) can be considered for gap acceptance. Thus,

$$\begin{aligned} q_{a,1} &= f_{a,4} (1 - \rho_1) q_1 \\ q_{a,4} &= f_{a,1} (1 - \rho_4) q_4. \end{aligned} \quad (3.85)$$

Streams 2, 3, 5, and 6 are assumed to be Poisson processes with parameters $\lambda_2 = q_2/3600$, $\lambda_3 = q_3/3600$, $\lambda_5 = q_5/3600$, and $\lambda_6 = q_6/3600$. During the gap acceptance periods streams 1 and 4 are assumed to have shifted exponential headway distributions with scale parameters $\lambda_1 = q_1/3600$ and $\lambda_4 = q_4/3600$ and location parameters equal to the follow-up times $t_{f,1}$ and $t_{f,4}$.

During the time periods available for gap acceptance Rank 2 streams are not blocked by Rank 1 vehicles, so that all streams 1, 2, 4, and 5 can be considered independent. The capacity of stream 8 can be obtained using equation (3.40) as

$$\begin{aligned}
 C_{m,8} &= (1 - \rho_1)(1 - \rho_4) \frac{q_M e^{(-q_M t_{c,8} + q_1 t_{f,1} + q_4 t_{f,4})/3600}}{1 - e^{q_M t_{f,8}/3600}} \\
 &= \frac{(1 - \rho_1)(1 - \rho_4)}{e^{-q_1 t_{f,1} - q_4 t_{f,4}}} \times \frac{q_M e^{-q_M t_{c,8}/3600}}{1 - e^{q_M t_{f,8}/3600}} \\
 &= \boxed{f_{1 \wedge 4} \frac{q_M e^{-q_M t_{c,8}/3600}}{1 - e^{q_M t_{f,8}/3600}}},
 \end{aligned} \tag{3.86}$$

where $q_M = q_1 + q_2 + q_4 + q_5 + q_6$. Likewise, the movement capacity of minor stream 11 is

$$C_{m,11} = \boxed{f_{1 \wedge 4} \frac{q_M e^{-q_M t_{c,11}/3600}}{1 - e^{q_M t_{f,11}/3600}}}, \tag{3.87}$$

where $q_M = q_1 + q_2 + q_3 + q_4 + q_5$. If $\rho_1 \geq 1$ or $\rho_4 \geq 1$, Rank 3 capacities are null.

In the calculation of q_M the major road through flows are adjusted for the possible effects of nonconflicting right-turn streams (stream 3 on stream 8 and stream 6 on stream 11). However, in the estimation of Rank 2 capacities this adjustment is not made, but the effect of major road right-turn streams is considered directly.

3.3 Capacity of Rank 4 streams

Rank 4 streams 7 and 10 (left turns from minor road) in four-legged intersections have three higher priority streams. A Rank 4 vehicle can enter the intersection when there is an acceptable gap in the priority streams and no conflicting Rank 2 or Rank 3 queues. These queues are, however, not independent. The queues of a Rank 3 stream can discharge when there is a gap larger than the critical gap in Rank 1 and Rank 2 priority streams, and conflicting Rank 2 queues are empty. Because Rank 3 queues depend on the state of Rank 2 queues, the probability of empty Rank 2 and Rank 3 queues cannot be obtained by a simple multiplication.

For Rank 4 stream 7 (10) the conflicting Rank 3 stream is the opposing through stream 11 (8). Conflicting Rank 2 streams are the major road left turning streams 1, 4 and the opposing minor road right turning stream 12 (9). The latter stream has no effect on the Rank 3 queues in stream 11 (8). If the major road has separate lanes for entering streams 7 (10) and 12 (9), the streams do not have a conflict.

Because of the mutual dependency of queues, conflicting Rank 2 and Rank 3 queues occur more simultaneously than would be the case, if the queuing pro-

cesses were independent. Ignorance of this dependency would give too low capacity estimates. Brilon and Großmann (1991) have developed by simulation a corrected probability:

$$p_{z,i} = 0.65p_{0,1 \wedge 4 \wedge i} - \frac{p_{0,1 \wedge 4 \wedge i}}{p_{0,1 \wedge 4 \wedge i} + 3} + 0.6\sqrt{p_{0,1 \wedge 4 \wedge i}}, \quad (3.88)$$

where $p_{z,i}$ is the probability of empty queues in major road left-turn streams 1, 4, and minor road through stream i , and $p_{0,1 \wedge 4 \wedge i} = p_{0,1}p_{0,4}p_{0,i}$ (see Grossmann 1991, Kyte et al. 1996). This equation is also used in HCM2000 (Transportation Research Board 2000). As Figure 3.20 displays, the corrected probability (thick curve) is larger than the simple product $p_{0,1 \wedge 4 \wedge i}$ (diagonal line). The movement capacity of streams 7 and 10 is calculated as

$$\begin{aligned} C_{m,7} &= p_{z,11}p_{0,12}C_{p,7} \\ C_{m,10} &= p_{z,8}p_{0,9}C_{p,10}. \end{aligned} \quad (3.89)$$

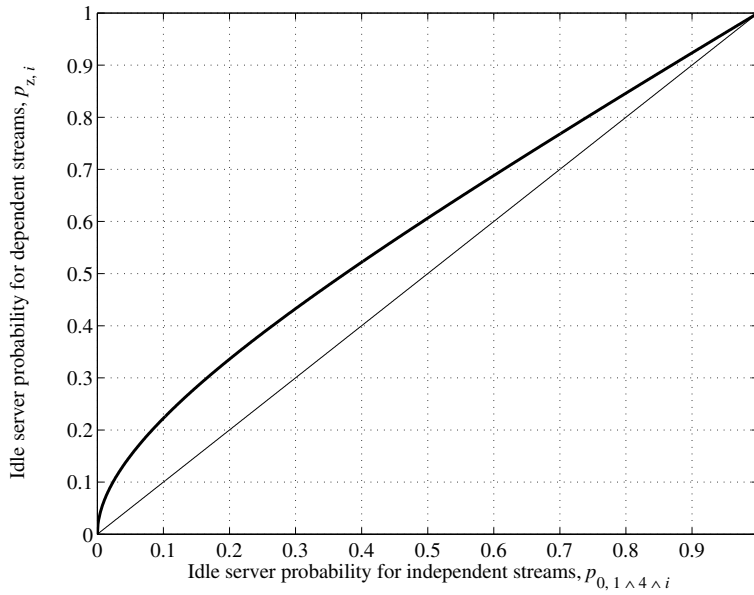


FIGURE 3.20 · Corrected probability of empty queues in streams 1, 4, and i (8 or 11) (Brilon and Großmann 1991)

The German HCM (Forschungsgesellschaft für Straßen- und Verkehrswesen 2001) uses a different approach:

$$p_{z,i} = \frac{1}{1 + \frac{1 - p_{0,1}p_{0,4}}{p_{0,1}p_{0,4}} + \frac{1 - p_{0,i}}{p_{0,i}}}. \quad (3.90)$$

The equation can also be expressed as

$$p_{z,i} = \frac{p_{0,1 \wedge 4 \wedge i}}{p_{0,1 \wedge 4} + p_{0,i} - p_{0,1 \wedge 4 \wedge i}}, \quad (3.91)$$

which can be interpreted as the probability of an idle server in all streams 1, 4, and i under the condition that a server is known to be idle either in both left-turn streams 1 and 4 or in stream i . All service processes are assumed to be independent. The HCM2000 equations (3.89) are used to obtain the movement capacities of streams 7 and 10.

Figure 3.21 displays the German impedance factor against different values of $p_{0,1 \wedge 4 \wedge i}$ and $p_{0,i}$. When there is no traffic in major road left-turn streams ($p_{0,1 \wedge 4} = 1$), the probability of a queue-free state in all priority streams is $p_{0,i}$, as expected. As Figure 3.22 displays, the HCM2000 impedance factor is not consistent under these conditions. The same observation can be made, when there is no traffic in stream i ; i.e., $p_{0,i} = 1$.

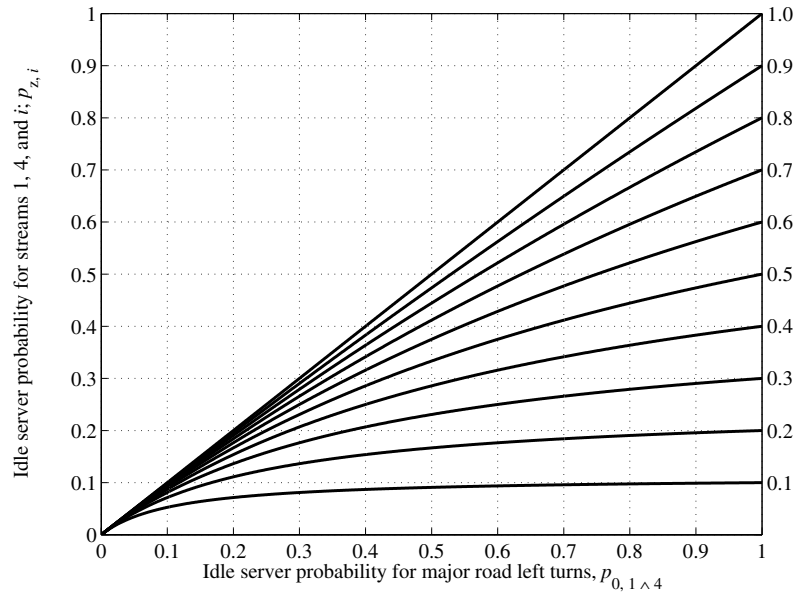


FIGURE 3.21 · German impedance factor $p_{z,i}$ with different values of $p_{0,1 \wedge 4}$ and $p_{0,i}$ (right) (Forschungsgesellschaft für Straßen- und Verkehrswesen 2001)

Brilon and Wu (2002) described these approaches as pragmatic simplifications, which are only of an approximative nature. They chose to propose an even simpler approximation, which is easier to use in practical applications. However, below is described a theoretically more solid approach.

The potential capacity $C_{p,k}$ of a Rank 4 stream k is given by equation (3.9), where the major flow rate q_M is the sum of conflicting higher priority streams. Following equation (3.74) this capacity should be adjusted for the utilization of server capacity and modified headway distributions of higher priority streams.

First it is necessary to define a utilization factor of a service system with two streams (i and j) of units (vehicles). The average arrival rates are λ_i and λ_j , and average service times x_i and x_j . The utilization factor is the product of average

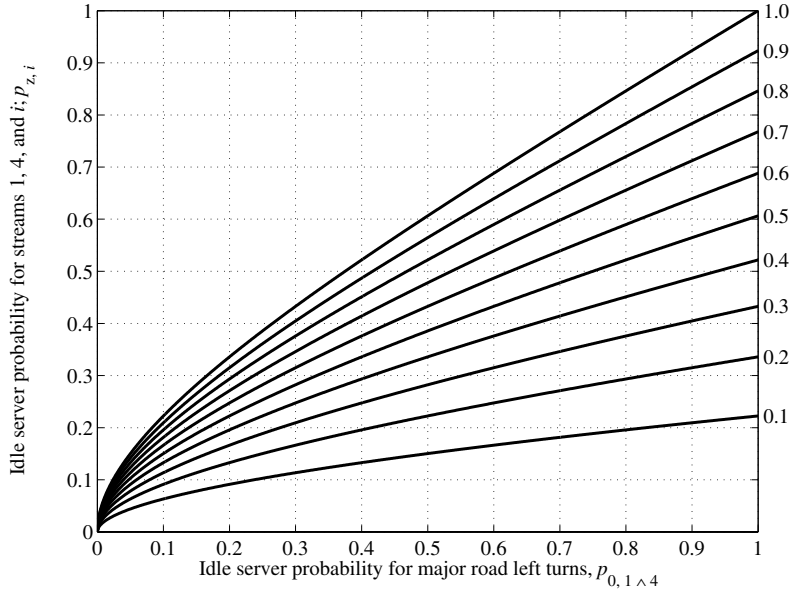


FIGURE 3.22 · HCM2000 impedance factor $p_{z,i}$ with different values of $p_{0,1 \wedge 4}$ and $p_{0,i}$ (right)

arrival rate and the average service time. The average arrival rate of units is $\lambda_i + \lambda_j$ and the average service time is the weighted average

$$x = \frac{\lambda_i x_i + \lambda_j x_j}{\lambda_i + \lambda_j}. \quad (3.92)$$

The utilization factor of the service system is

$$\rho_{i,j} = (\lambda_i + \lambda_j)x = \lambda_i x_i + \lambda_j x_j = \rho_i + \rho_j, \quad (3.93)$$

which is the sum of the utilization factors of the component streams.

Let us now assume that stream i has Rank 2 and stream j has Rank 3. The arrival rates are $q_i = 3600\lambda_i$ and $q_j = 3600\lambda_j$. Rank 2 stream i has follow-up time $t_{f,i}$ and average service time $x_i = 3600/C_{m,i}$. The utilization factor is $\rho_i = \lambda_i x_i$. Rank 3 stream j has follow-up time $t_{f,j}$ and movement capacity $C_{m,j}$. In this case the utilization factor ρ_j is different than the v/c ratio $\rho'_j = q_j/C_{m,j}$. Because stream j vehicles are served only during the fraction $1 - \rho_i$ of time, the average service time is $x_j = 3600(1 - \rho_i)/C_{m,j}$ and the utilization factor $\rho_j = \lambda_j x_j = (1 - \rho_i)\rho'_j$. This gives an interesting interpretation for the v/c ratio of a Rank 3 stream:

$$\rho'_j = \frac{\rho_j}{1 - \rho_i}. \quad (3.94)$$

The v/c ratio of a Rank 3 stream j gives the probability that a stream j vehicle is being served (waiting for a gap or a follow-up time) given that there are no Rank 2 vehicles in the service system.

The utilization factor of the total service system is

$$\rho_{i,j} = \rho_i + \rho_j = \rho_i + (1 - \rho_i)\rho'_j = \rho_i + \rho'_j - \rho_i\rho'_j, \quad (3.95)$$

which is the probability that there is a vehicle in the service system. The probability of an empty system is now obtained directly as

$$1 - \rho_{i,j} = 1 - [\rho_i + (1 - \rho_i)\rho'_j] = (1 - \rho_i)(1 - \rho'_j). \quad (3.96)$$

The queuing processes are, however, not independent, but the effect of stream i on the queuing process of stream j is carried in the v/c ratio ρ'_j , as demonstrated above (3.94). For a four-leg intersection this indicates that

$$p_{z,i} = p_{0,1 \wedge 4} p_{0,i} = p_{0,1} p_{0,4} p_{0,i}. \quad (3.97)$$

This is displayed in figure 3.23 and as the diagonal line in figure 3.20. It should, however, be observed that the model discussed here includes also a capacity adjustment for the headway distribution of Rank 2 and Rank 3 streams, which adjustment is not included in HCM2000. In addition, equation (3.54) gives higher capacities and lower utilization factors than HCM2000 for Rank 3 streams, which increases the Rank 4 capacity.

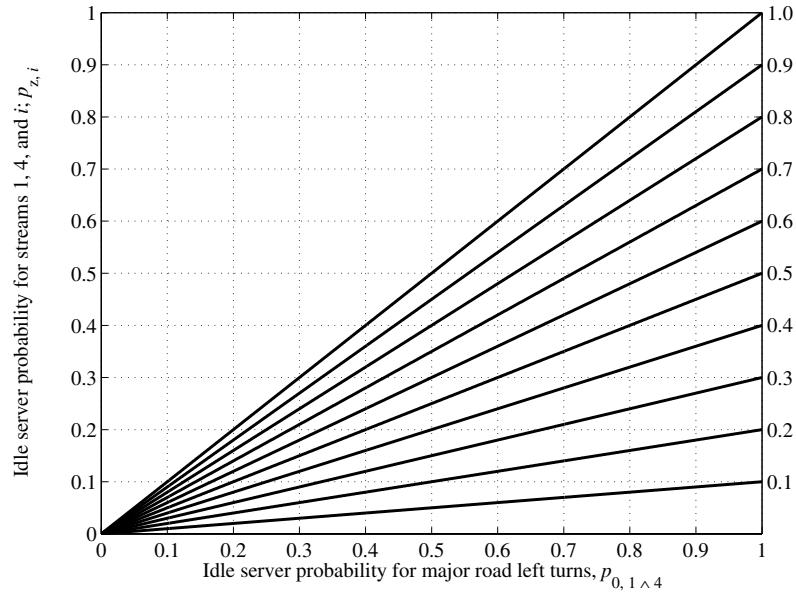


FIGURE 3.23 · Linear impedance factor $p_{z,i}$ with different values of $p_{0,1 \wedge 4}$ and $p_{0,i}$ (right)

The discussion above gives an important result: Capacities of Rank 2, 3, and 4 streams are calculated in a hierarchical order. The capacities of higher priority streams are used as input in the capacity estimation of lower priority flows. These

capacity estimates contain all the information necessary about the hierarchy of streams. *When the capacities of major streams are known, their relative priorities can be ignored in the capacity estimation of a minor flow.* If q_M is the sum of flow rates of all major streams and μ is the set of all major flows of Rank 2 or higher (i.e., lower priority), the movement capacity of a minor stream k can be expressed as

$$C_{m,k} = \frac{\prod_{i \in \mu} p_{0,i}}{e^{-\sum_{i \in \mu} q_i t_{f,i}/3600}} C_{p,k}, \quad (3.98)$$

where $C_{p,k}$ is the potential capacity of stream k assuming that all major streams are independent, $p_{0,i} = (1 - \rho'_i)_+$, and ρ'_i is the v/c ratio of stream i . This equation can be used to estimate the movement capacity of both Rank 3 and Rank 4 streams. Figures 3.24 and 3.25 display the movement capacities of equation (3.98) compared with Monte Carlo simulations. Figures 3.26 and 3.27 show that equation (3.98) gives higher capacities than HCM2000.

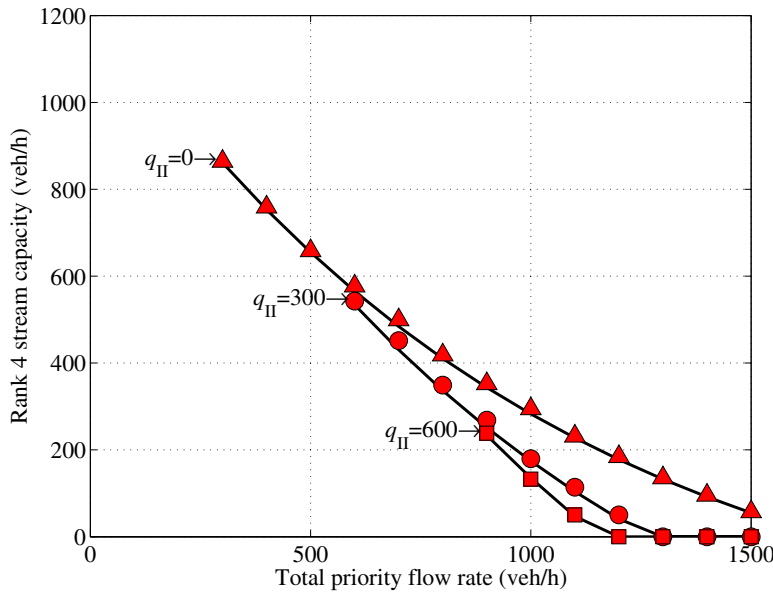


FIGURE 3.24 · Movement capacity (3.98) of a Rank 4 stream when Rank 3 flow rate is $q_{III} = 300$ veh/h compared with Monte Carlo simulations (markers)

The movement capacities of streams 7 and 10 are

$$C_{m,7} = \frac{p_{0,1} p_{0,4} p_{0,11} p_{0,12} C_{p,7}}{e^{-(q_1 t_{f,1} + q_4 t_{f,4} + q_{11} t_{f,11} + q_{12} t_{f,12})/3600}} \quad (3.99)$$

$$C_{m,10} = \frac{p_{0,1} p_{0,4} p_{0,8} p_{0,9} C_{p,10}}{e^{-(q_1 t_{f,1} + q_4 t_{f,4} + q_8 t_{f,8} + q_9 t_{f,9})/3600}}, \quad (3.100)$$

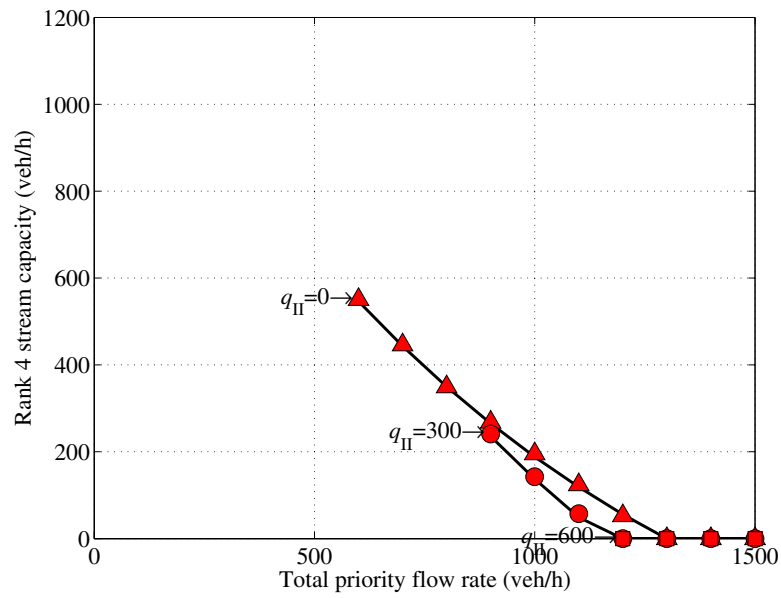


FIGURE 3.25 · Movement capacity (3.98) of a Rank 4 stream when Rank 3 flow rate is $q_{III} = 600$ veh/h compared with Monte Carlo simulations (markers)

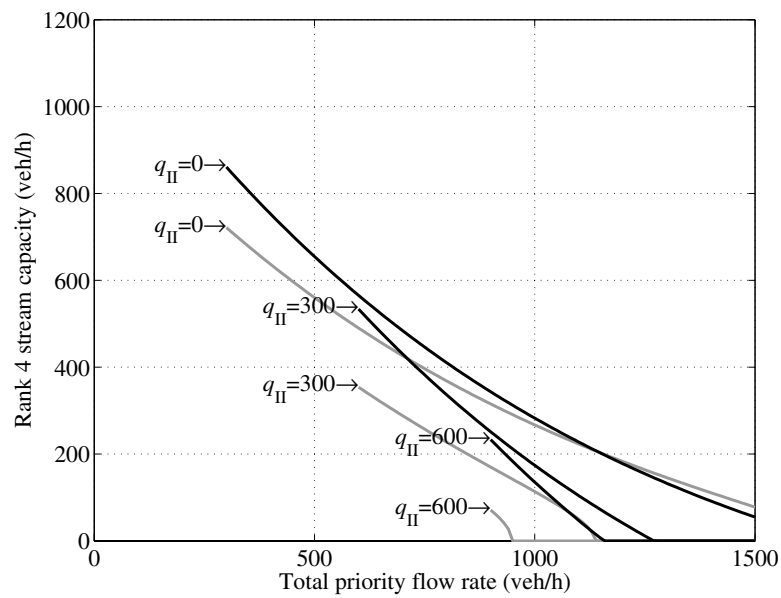


FIGURE 3.26 · Movement capacity (3.98) of a Rank 4 stream when Rank 3 flow rate is $q_{III} = 300$ veh/h (black curves) compared with HCM2000 (gray curves)

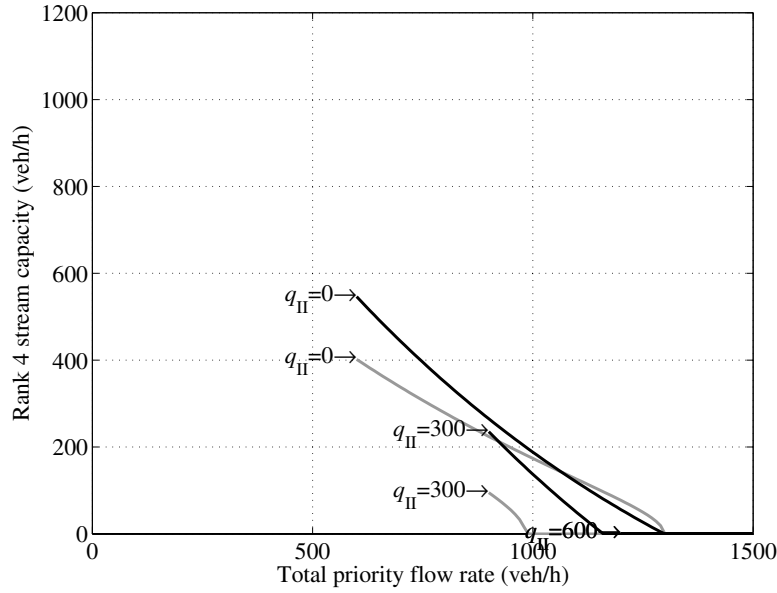


FIGURE 3.27 · Movement capacity (3.98) of a Rank 4 stream when Rank 3 flow rate is $q_{\text{III}} = 600$ veh/h (black curves) compared with HCM2000 (gray curves)

where $C_{p,k}$ is the potential capacity of stream k (7 or 10) assuming that all major streams are independent and have negative exponential headway distributions with scale parameter $\lambda_i = q_i/3600$. The possible effect of nonconflicting major road right turning streams (3 and 6) should be included in the major road through stream (2 and 5) flow rates.

3.4 Capacity of shared lanes

On shared lanes multiple streams are combined. If an approach in a four-leg intersection has only one lane, all three streams (left, through, and right) share the same lane. If the movement capacity of stream i is $C_{m,i}$, the average departure headway from a continuous queue is

$$h_i = \frac{1}{C_{m,i}}. \quad (3.101)$$

The average departure headway on a lane sharing n streams is

$$h = \sum_{i=1}^n p_i h_i = \sum_{i=1}^n \frac{p_i}{C_{m,i}}, \quad (3.102)$$

where p_i is the proportion of stream i on the lane. The shared lane capacity is (Harders 1968)

$$C_m = \frac{1}{h} = \frac{1}{\sum_{i=1}^n \frac{p_i}{C_{m,i}}} \quad (3.103)$$

Equation (3.103) can also be used to estimate the capacity of a stream consisting of different classes of vehicles with different gap-acceptance properties. Each vehicle class is considered as a separate stream. Brilon (1988*b*) used this approach to confirm the German passenger-car equivalencies presented by Jessen (1968).

4

Conclusions

The discussion above has presented a unified theoretical analysis of capacity at unsignalized intersections. First the gap acceptance theory was described, including the hierarchy of priorities at an unsignalized intersection and a queuing model of gap acceptance and queue discharge. The properties of the most important headway and lag distributions were presented.

Potential capacity equations for different headway and gap acceptance functions were presented following the results of earlier researchers. The capacity analysis for multiple independent major streams was based on the theory of superimposed point processes. It was possible to obtain a very general capacity equation, which could also be applied in the Rank 3 capacity analysis.

Gap acceptance and queue discharge of a Rank 3 stream was described by a queuing model. It demonstrated that in the current methodology the probability of a queue-free state in Rank 2 streams should be modified. In addition, the headway distribution during a queue-free state should have a minimum headway equal to the follow-up time, and the flow rate should be adjusted accordingly. If the arrivals to an intersection are Poisson processes, the headways of Rank 2 streams during the queue-free periods follow shifted exponential distribution. The modified movement capacity equations were presented. The resulting capacities were higher than the HCM2000 (Transportation Research Board 2000) capacities, especially at low traffic volumes.

In the analysis of Rank 4 capacity the dependency of Rank 3 queues on Rank 2 queues has been considered problematic. However, it was demonstrated that the v/c ratio of a Rank 3 stream j gives the probability that a stream j vehicle is being served (waiting for a gap or a follow-up time) given that there are no Rank 2 vehicles in the service system. Therefore, the v/c ratio contains the necessary information of this dependency. Thus, when the capacities of major streams are known, their relative priorities can be ignored in the capacity estimation of a minor flow. This makes the analysis much simpler, and the capacity equation for Rank 4 streams does not become too complicated. In fact, the same equation can be used to estimate the movement capacity of both Rank 3 and Rank 4 streams. The resulting capacities are higher than suggested by the HCM2000 method.

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